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INDIA METEOROLOGICAL DEPARTMENT

FORECASTING MANUAL

PART II

METHODS OF ANALYSIS

I: MAP PROJECTIONS FOR WEATHER CHARTS

BY

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FORECASTING MANUAL

Part II. Methods of Analysis

1. Map Projections for Weather Charts

by

K. Krishna

1. Introduction.

1.1 In order to represent the features on the surface of the terrestrial globe it will be most appropriate to have a globe of the same shape as the earth but reduced in scale. But obviously, this is often inconvenient; for, one cannot handle the globe with the same facility as the pages of an atlas. One cannot, for instance, have a globe for each synoptic chart. So it becomes necessary to represent the features of a globe on a plane (or flat) chart.

1.2 A map projection may be defined as the law of representing points on the surface of the earth by points on a plane surface. Conventionally, the points on the surface of the earth are represented by their latitudes and longitudes. Map projection is, therefore, a transformation of the latitude-longitude grid on the curved surface of the earth to a corresponding grid on the flat surface of a map by establishing a one-to-one correspondence between points. The law of transformation gives the lines on the map that stand for the latitudes and longitudes, that is to say, it orients the latitudes and longitudes of the projection. The latitude-longitude grid thus projected, is called the "graticule". When once the graticule that is characteristic of the projection is drawn, the geographical or other features can be drawn.

1.3 The process of map-making may be considered as consisting of two stages,

(a) projecting the surface of the earth on a developable surface* called the image surface and

(b) reducing the map obtained on the image surface (opened out into a plane, if it is not already a plane) to some suitable scale, say, $1:10^6$ or $1:10^7$.

1.4 In the present discussion the earth is taken as perfectly spherical in shape. During the process of projecting the earth upon the image surface, the surface of the earth undergoes varying degrees of expansion and contraction. If one takes the whole peel of an orange which can be assumed to be spherical in shape and tries to flatten it out into a plane surface, one will find either that there are gaps or there are overlapping creases or both. It is not possible to flatten a spherical surface, try as one may. In other words, the sphere is not a developable surface. It is not, therefore, possible to represent on one map all the features of the geographical elements like shape, area, bearing etc. Some of these properties are to be sacrificed in projection, retaining only those properties that are considered essential for the purpose in view. The properties chosen for preserving in projection determine the type of projection. There is hardly a projection that satisfies all the requirements.

2. Classification of Projections

2.1 All the different types of map projections can be divided into two broad categories:

(i) Geometric or Perspective projections and

(ii) Non-geometric or non-perspective or mathematical projections.

The geometric projections are obtained as shadows of the earth cast on the image surface by rays of light emanating from a fixed or variable point. In the non-perspective projection this simple visualization does not hold good. Within these two broad classes a variety of projections may be obtained,

(i) by choosing the property to be preserved in the projection

and (ii) by varying the surface on which the projection is made.

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* Certain curved surfaces like a cone, a cylinder or a hyperboloid of revolution are such that they can be completely covered by (families of) straight lines which can be drawn on them. These straight lines are called "generators" of the surfaces. Of these, surfaces like cone and cylinder, are such that if the curved surface is cut along a generator, it can be opened out into a plane (flat) surface. Such surfaces are called "Developable Surfaces".

2.2 The properties that can be preserved in projection are map scale, area or bearing. The projections based on the preservation of these properties are as follows:

(a) Conformal Projection or Orthomorphic Projection: In this, at any point, the map scale is the same in all directions. This property means that angles are preserved in projection and, therefore, forms or shapes of small areas remain the same. This property is called conformality or orthomorphism. Conformal maps are used for synoptic charts because one wants the shape of isobars, contours and other similar lines to remain the same as on the surface of the earth.

(b) Equal Area or Equivalent or Authalic Projection: Here equal areas anywhere on the globe are represented by equal areas on the map; that is, areas are preserved. Equal area maps are useful for certain climatological charts where areas in different parts of the globe require to be compared.

(c) Azimuthal Projection: Bearings from the centre of the map to any point remain the same as on the earth's surface, in this projection.

2.3 The projections can also be classified according to the nature of the surface on which the projection is made. They are:-

(a) Zenithal Projection in which the projection is made on a plane surface,

(b) Cylindrical Projection in which the projection is made on the surface of a cylinder and

(c) Conical Projection in which the projection is made on the surface of a cone.

2.4 Combinations of the property to be preserved, the surface of projection and the point of origin of the projection lead to a variety of projections. However, it must be stated that only certain combinations are possible.

3. Geometric Projections

3.1 In the geometric projection the source of light (or the point of origin of projection) can be varied in relation to the earth-sphere, leading to projections having different properties (Fig. 1a).

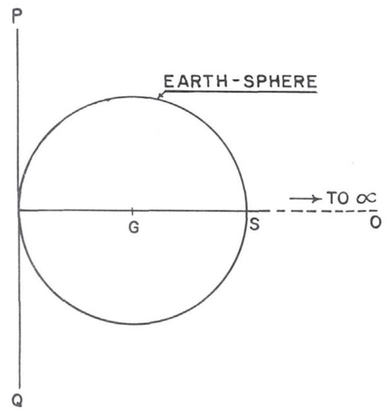

FIG. 1 (a) GEOMETRIC PROJECTIONS - Positions of centres of projections

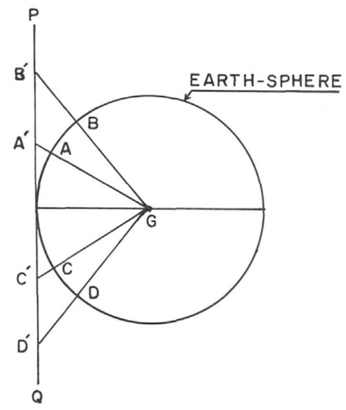

FIG. 1 (b) GNOMONIC PROJECTION

PQ is the plane of projection.

G,S and O (at infinity) are the points of origin of Gnomonic, Stereographic and Orthographic projections respectively.

B', A', C' and D' are the images (projections) of points B, A, C and D respectively on earth's surface.

3.2 The projection of the earth from its centre on a plane surface touching the earth at any point, (which is the projection on a tangent plane obtained by placing a source of light at the centre of the earth) is called the Gnomonic Projection (Fig. 1b). In this projection, Great Circles* project into straight lines formed by the intersection of the plane of projection by the plane of the Great Circle. This property of the Gnomonic Projection is of great value for navigation, as the Great Circle route (which is the shortest route) between two points on the surface of a sphere, is easily obtained as the straight line joining the two points on the map. However, the maximum of the earth's surface which can be projected in this type of projection will, at best, be a hemisphere. See Fig. 1c for a graticule of the Gnomonic Projection.

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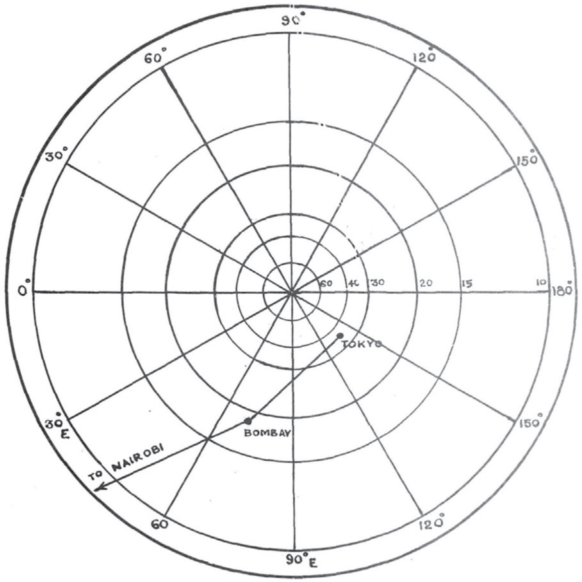
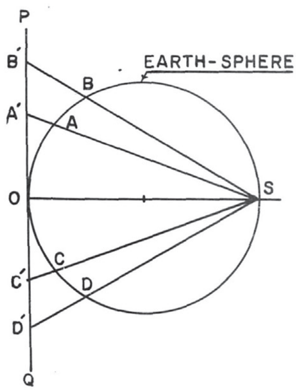
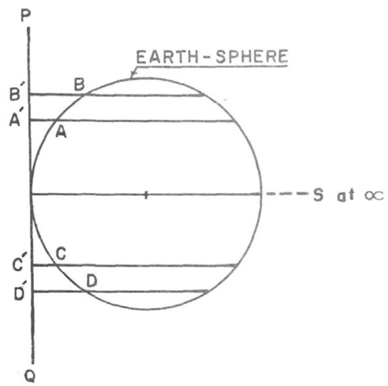
* Great Circle - A plane whose distance from the centre of the sphere is less than its radius, intersects it along a circle whose radius is, in general, less than the radius of the sphere. If, however, the plane passes through the centre of the sphere, the intersecting circle is the largest of its kind with radius equal to the radius of the sphere. Such a circle is called a "Great Circle", the others "Small Circles".



Note that the distance between latitudes increases rapidly towards equator
Great circle route between two places is indicated by the straightline joining the two places on the map.

FIG. I (c) THE GRATICULE OF POLAR GNOMONIC PROJECTION





S - Point of Origin of Projection; PQ - Plane of Projection;

B',A',C'and D' are the images (project~~ed~~ions) of points B,A,C and D respectively on earth's surface.

FIG. I(d) STEREOGRAPHIC PROJECTION FIG. I(e) ORTHOGRAPHIC PROJECTION

3.3 If the source of light is placed at a point on the surface of the sphere and the shadow obtained on a plane tangent to the sphere at the diametrically opposite point, we get the Stereographic Projection (Fig. 1d). The Stereographic Projection is also a conformal projection as will be seen later. If the source of light is at infinity and the plane of projection is normal to the rays of light, we get the Orthographic Projection (Fig. 1e).

4. Meaning of Conformality

4.1 As conformal projection is widely used for synoptic charts this projection will be dealt with in some detail. The scale of the image map or the image scale at a point in a given direction, is defined as the limit of the ratio of the distance between that point and its neighbour in the given direction and the distance between the corresponding points on the surface of the earth. This ratio may, in general, be different in different directions. Conformal projection is defined as one in which the ratio is the same in all directions. If A, B and C are three points on the surface of the earth and A', B' and C' are their projections, then in the limit of the area of the triangle ABC becoming zero, the ratios of the corresponding sides of the two triangles ABC and A'B'C' will be equal since by the definition, the scale is the same in all directions. Therefore, the triangles are similar and the corresponding angles are equal. Hence, in general, the angle between any two lines on the surface of the earth is the same as the angle between the projections of these lines. That is, the magnitudes of angles are preserved in projection. This property is responsible for the preservation of shapes of small regions. Hence the name given for this class of projections is "Conformal" (of the same from) or "Orthomorphic" (of the correct form).

4.2 Conformality holds good in the mathematical sense, that is in the limit; it is true only over infinitesimal areas. When the area is very large, the shape of an area on the earth's surface is not the same as the shape of its projection. For small areas, the difference in shape will not be significant. A rectangle bounded by straight lines can be conformal with a rectangle bounded by curved lines if the angles at the corners are the same. However they will appear similar only if the area is small. The shape may differ and the difference becomes perceptible if the area covered is very large.(See Fig. 2a). The greater the length of the lines the greater will be the distortion. As an extreme case, the triangle bounded by two meridians inclined at an angle $\lambda°$ and the Equator, will project into a rectangle of limitless length in the Mercator, a sector of circle of angle $n\lambda°$ in the Lambert's Conformal Conical and a sector of circle of angle $\lambda°$ in the Polar Stereographic Projections. It may be noted here that the Pole is a point of singularity where the condition of conformality is not always satisfied (see Fig. 2b).

4.3 While plotting a scalar on a map there is a one-to-one correspondence between the point in the map and the point on the earth's surface and no representation of a direction is needed. While representing a vector, like wind, on a map, the direction of the vector is to be represented correctly. A conformal map simplifies the plotting of a vector since there is no difference in the direction on the map and the direction on the earth.

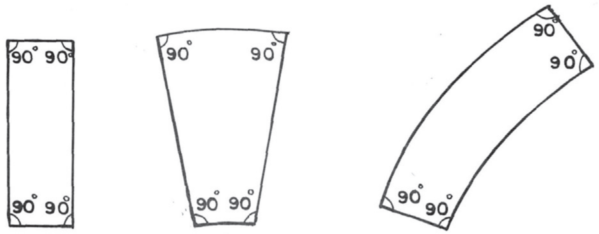
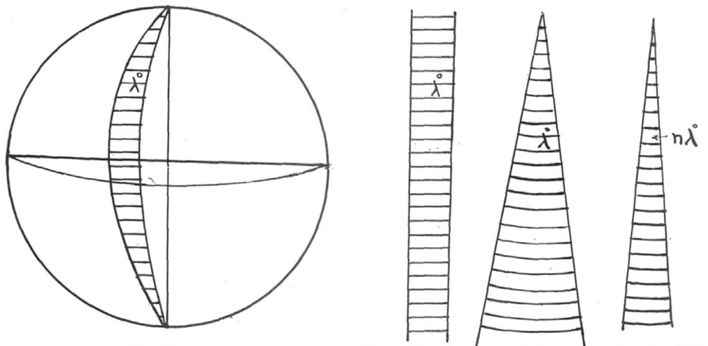


FIG. 2 (a)

These three figures are Conformal projections of one another, although they look dissimilar.



Earth

Mercator's Projection.

Polar Stereographic Projection.

Lambert's Conformal Conical Projection.

FIG. 2 (b)

FIG. 2: THE MEANING OF CONFORMALITY

Projection of a meridional strip in the three different Conformal projections. λ is the angle between two meridians, and n is the constant of the cone.

5. Conformal Projections

5.1 There are three conformal projections used for weather charts – Mercator's Projection, Lambert's Conformal Conical Projection and Polar Stereographic Projection. Of these projections, the first two are non-perspective, while the third one is perspective.

5.2 If the surface chosen for projection is a cylinder with its axis coinciding with the axis of the earth, one obtains the Mercator's Projection. This was introduced in 1559 by G. Mercator. The radius of the cylinder is still variable and can be chosen conveniently so as to make the scale true at any specified latitude. Before reduction of scale, if one can imagine a cylinder whose radius is equal to the radius of the earth, then it is obvious that the cylinder will touch the earth along the Equator. The projected length of the Equator will be same as on the surface of the earth; that is, the scale ratio is one at the Equator. Alternately, it is also expressed that scale is "true" along the Equator or the Equator is the Standard Parallel. This projection is called the Tangent Mercator's Projection. On the other hand, if the radius of the cylinder is smaller than the radius of the earth, the cylinder will not touch but will intersect the earth (sphere) along two latitude circles placed symmetrically on either side of the Equator, any, $\varphi°$ North and $\varphi°$ South. This projection is called Secant Mercator's Projection or Mercator's Projection with two standard Parallels (Fig. 3).

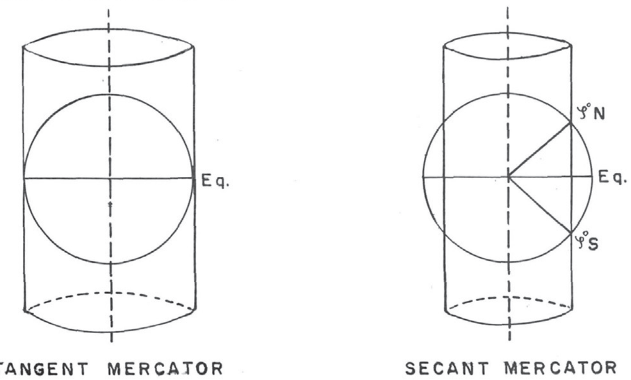
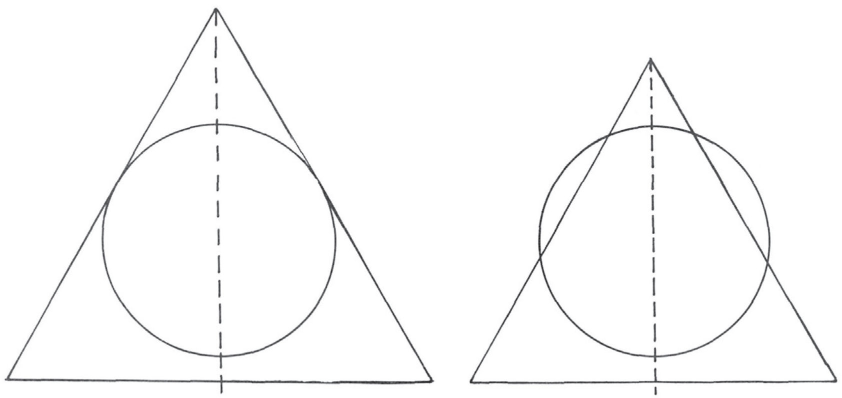


FIG. 3: MERCATOR'S PROJECTION

The surface of projection is a cylinder.

5.3 If the image surface is a cone with its axis coinciding with the axis of the earth we obtain Lambert's Conformal (Orthomorphic) Conical Projection. This was introduced by J.G. Lambert in 1772. As in the case of the Mercator's Projection, the cone may touch or intersect the earth (sphere). Since the axis of the cone lies along axis of the earth, in the former case the cone will touch the earth along a latitude circle. The scale will be true along this latitude which is then called the Standard Parallel. As the cone touches the earth this projection is also called the Tangent Conical Projection. On the other hand, the cone may pierce through the earth along two parallels of latitude φ_1 and φ_2 . In this case the scale will be true along these two latitudes both of which now constitute the Standard Parallels. This projection is known as the Lambert's Conformal (Orthomorphic) Conical Projection with two Standard Parallels φ_1 and φ_2 . It is also sometimes called Secant Conformal Conical Projection with the cone cutting the sphere along parallels φ_1 and φ_2 (Fig. 4).

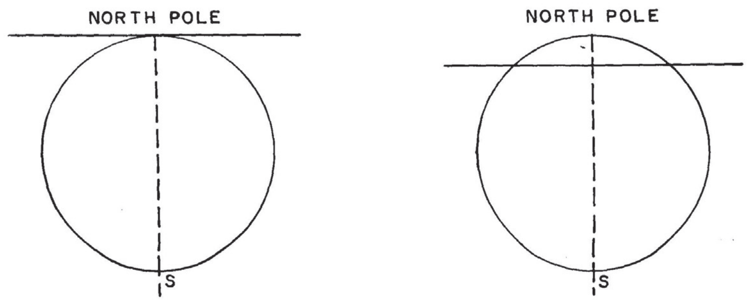


TANGENT CONICAL PROJECTION SECANT CONICAL PROJECTION

FIG.4 : LAMBERT'S CONICAL PROJECTION

The surface of projection is a cone.

5.4 If the image surface chosen is a plane perpendicular to the axis of the earth we get the Polar Stereographic Projection. This was introduced in the first half of 18th century. This is a perspective projection. Here one can get the latitude-longitude lines on the surface of projection as images of the latitude-longitude circles on the earth's surface, cast by a source of light placed at the Pole farther away from the surface of projection. In this case also, as in the two earlier cases, the plane may touch the earth at the Pole or cut it along any latitude. In the former case the scale is everywhere more than one except at the point representing the Pole. If it cuts the earth at latitude φ_o the scale will be "true" at this latitude (Fig. 5). These are called respectively Polar and Circumpolar Stereographic Projection with φ_o as the Standard Parallel. However, in practice, the term "Polar Stereographic Projection" is used to cover both.



POLAR STEREOGRAPHIC PROJECTION CIRCUM-POLAR STEREOGRAPHIC PROJECTION

FIG.5 : STEREOGRAPHIC PROJECTION

The surface of projection is a plane.

Point of origin of projection is south pole.

5.5 These three projections belong, in essence, to the same class because the Mercator's and Polar Stereographic Projections are extreme cases of Lambert's Conformal Conical Projection. If one goes on decreasing the angle of the cone by shifting the apex of the cone along the axis of the earth farther and farther away from the centre of the earth, the latitude circle of contact of the earth and cone goes on approaching the Equator. In the limit, when the apex is at infinity the cone becomes a cylinder touching the earth along the Equator. We, then, get the Mercator's Projection. If the cone cuts the earth, we can imagine that the apex is taken to infinity keeping the upper latitude circle of intersection unchanged. Then the lower latitude circle will slide down the Equator. When, eventually, the apex is at infinity one gets the cylinder cutting the earth along circles of equal latitude on either side of the Equator. This leads to Secant Mercator's Projection.

Instead of sliding the apex of the cone up, if one slides it down the axis of the earth, the latitude circle of contact goes higher and higher up, and, in the limit, the cone becomes a plane touching the earth at the Pole. One, thus, gets the Tangent Polar Stereographic Projection. In the case of the secant cone, if one keeps one of the latitude circles of intersection the same and brings down the apex one gets, in the limit, a plane cutting the cone along the fixed latitude circle. In this case, one obtains the Circumpolar Stereographic Projection with the plane of projection cutting the earth along a Standard Parallel.

5.6 The World Meteorological Organization has, with a view to securing uniformity in practice, recommended for synoptic weather maps, the use of conformal projections suitable for different regions of the world, vide "Chapter 7 : Synoptic and Forecasting Practices" of W.M.O. Technical regulations Vol. 1 and Chapter 1 : WMO No. 151 TP.71 , Guide to the preparation of synoptic weather charts and diagrams. An extract of the recommendations is given in Appendix 3.

6. The Mercator's Projection

6.1 In the Mercator's Projection (Fig. 6), all the latitude circles project into horizontal circles on the cylinder, which, when the cylinder is opened out, become horizontal lines of identical length equal in value to the circumference of the cylinder. The meridians, on the other hand, project into vertical equally spaced lines cutting the latitude lines at right angles. The Cartesian coordinate system is convenient in discussing this projection. The latitude lines are parallel to x - axis and the longitude lines to y - axis. If the scale is true along latitude φ_o and the ordinate $x = 0$ represents the meridian $\lambda = 0$, then the abscissa of any meridian λ is,

$$x = a \lambda \cos \varphi_o \text{ , where } a \text{ is the radius of the earth;}$$

hence

$$\frac{\partial x}{\partial \lambda} = a \cos \varphi_o \text{ .} \qquad \text{...(6.1)}$$

The differential equation expressing the condition of conformality (vide Appendix 2), gives

$$\frac{\partial y}{\partial \varphi} = \sec \varphi \, \frac{\partial x}{\partial \lambda} \text{ ,}$$

that is

$$\frac{\partial y}{\partial \varphi} = a \cos \varphi_o \sec \varphi . \qquad \text{...(6.2)}$$

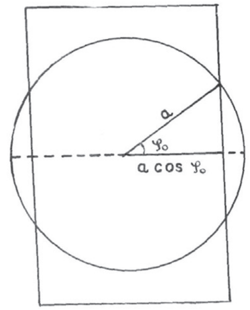


FIG. 6(a)

Vertical cross-section of cylinder in relation to the sphere.

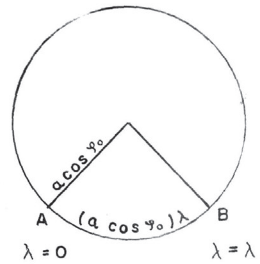


FIG. 6(b)

Horizontal cross-section of cylinder.

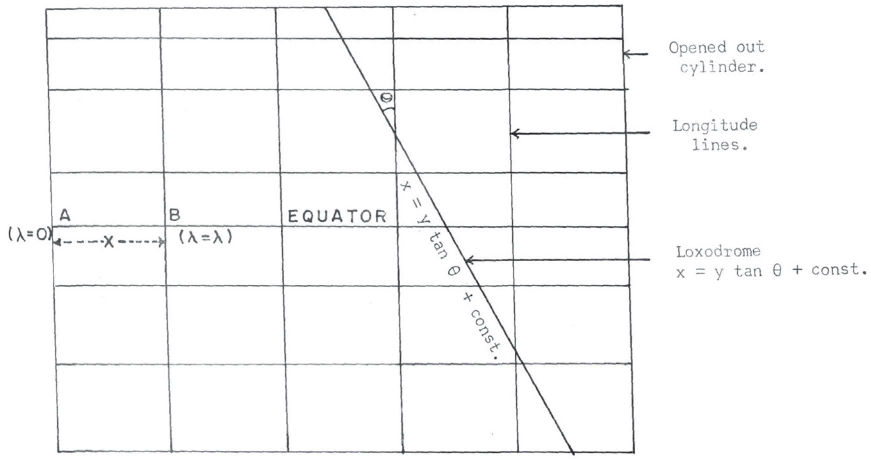


FIG. 6 (c)

Graticule of Mercator's Projection.

FIG. 6 : MERCATOR'S PROJECTION

On integrating,

$$y = a \cos \varphi_o \log \tan \left(\frac{\pi}{4} + \frac{\varphi}{2}\right) \qquad \text{...(6.3)}$$

taking the Equator as the x - axis (i.e. $y = 0$).

In the case of Tangent Mercator's Projection, since $\varphi_o = 0$,

$$y = a \log \tan \left(\frac{\pi}{4} + \frac{\varphi}{2}\right) \qquad \text{...(6.4)}$$

6.2 From the above equation, it is clear that on the map, distances of the latitude lines (from the Equator) increase rapidly and that either Pole is at an infinite distance from the Equator. The Poles can never be represented on a Mercator's Projection. Where polar regions are not important and do not require to be represented, the simple Mercator's map can be used.

6.3 The scale 'm' of the Mercator's Projection at any latitude is given by

$$m = \frac{\text{length of projected parallel at the latitude } \varphi}{\text{length of parallel on the globe}}$$

$$= \frac{2\pi a \cos\varphi_o}{2\pi a \cos\varphi}$$

$$= \frac{\cos\varphi_o}{\cos\varphi} = \cos\varphi_o \sec\varphi \,. \qquad \ldots(6.5)$$

From this equation, it can be seen that the scale is unity at latitude φ_o North or South. As one proceeds towards the Poles from latitude φ_o, $\cos\varphi$ falls more and more short of $\cos\varphi_o$ and hence the scale value exceeds unity more and more, becoming infinity at the Poles. Between latitudes φ_oN and φ_oS, $\cos\varphi$ exceeds $\cos\varphi_o$ more and more as one proceeds towards the Equator, making the scale increasingly less than unity and taking it to the minimum value of $\cos\varphi_o$ at the Equator. The values of the scale at various latitudes are given in Appendix 6. In the case of Tangent Mercator's Projection the scale is unity at the Equator and increases as one goes away from the Equator. In the Mercator's Projection every parallel is, in effect, stretched $\sec\varphi$ times its true length on the surface of the earth. The distances of these parallels from the Equator given by equation (6.3), are so adjusted as to make the scale along the meridian at any point equal to the scale along the parallels at the same point. In other words the inevitable east-west stretching is accompanied by an equal north-south stretching at every point over the entire projection.

6.4 The Mercator's Projection is symmetric about the Equator. It is most suitable for representing Equatorial or Tropical regions. It can also be used for securing continuous representation of the whole of the Northern and Southern hemispheres except the Polar regions where the scale is inordinately exaggerated. For example, Greenland, which is only one tenth of South America in area appears almost as large.

6.5 The most notable use of Mercator's Projection is in navigation on account of the fact that loxodromes or rhumb lines* on the surface of the earth are represented by straight lines in this projection. Navigators often find it convenient to navigate with a constant bearing to reach the destination. Mercator's chart helps them to find this bearing.

If the bearing of the loxodrome be θ, then,

$$\tan\theta = \frac{\text{component of the small interval travelled along the latitude circle on the earth}}{\text{corresponding component along the meridian}}$$

$$= \frac{a\cos\varphi\, d\lambda}{a\, d\varphi} \,.$$

Therefore

$$d\lambda = \tan\theta \sec\varphi \, d\varphi \,.$$

Since the bearing θ is constant, $\tan\theta$ is a constant.

On integration

$$\lambda = \tan\theta \log\tan\left(\frac{\pi}{4} + \frac{\varphi}{2}\right) + C \,.$$

On substituting for λ and $\log\tan\left(\frac{\pi}{4} + \frac{\varphi}{2}\right)$, their values [eq.(6.1)and(6.3)], in the Mercator's Projection,

$$x = y\tan\theta + \text{constant},$$

which is an equation for a straight line in the Cartesian Coordinates used in discussing the Mercator's Projection.

6.6 Another merit of the Mercator's Projection is the convenience in constructing it. Choose the scale of the map to suit the size of the chart. Draw the Equator as a straight line at the centre. Calculate the distance between the meridians using eq. (6.1). Divide the Equator in units of this distance and construct vertical lines which represent meridians. Compute the distances of latitude circles from the Equator from eq. (6.3) for suitably chosen latitudes or obtain them from tables and mark off these distances along any meridian. The latitudes will be parallel to the Equator through these points. Meridians and parallels for intermediate values also can be easily drawn. After constructing the graticule, preparing the map will be

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* Loxodrome or Rhumb Line is a line on the earth's surface (or its projection) making a constant angle with the meridian.

as easy as plotting a rectangular graph. A world map on Mercator's Projection, with typical Great Circle and Rhumb Line routes shown on it, is given in Fig. 7.

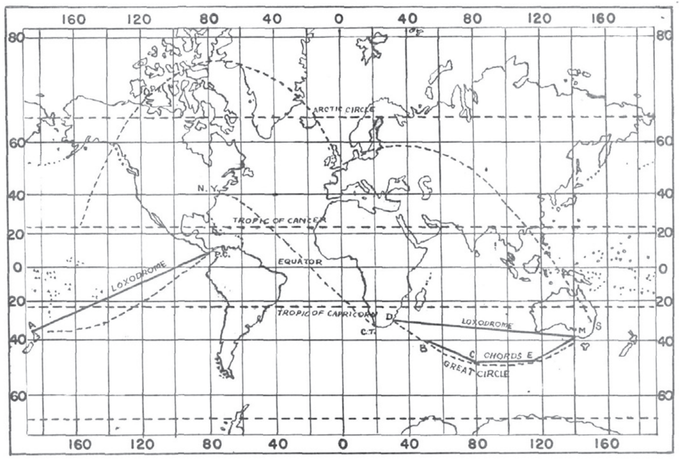

FIG. 7: MERCATOR'S PROJECTION SHOWING GREAT CIRCLES AND LOXODROMES

[From 'An Introduction to the Study of Map Projections' by J.A. Steers 1965 (Fourteenth Edition)]

Loxodromes are straight (unbroken) lines and great circles, curved (broken) lines. Note that the latter are convex to the nearer Pole. The Durban to Melbourne route is made up of several loxodromes very nearly approaching the great circle route.

7. The Lambert's Conformal (Orthomorphic) Conical Projection.

7.1 In the case of Lambert's Conformal Conical Projection (Fig. 8), the surface of projection is a cone intersecting the sphere along the two parallels φ_1 and φ_2, called the Standard Parallels (Fig. 8a). If the cone is opened out along a Generator*, we get a sector of a circle. The whole world can be mapped within this sector. The ratio the angle of this sector bears to 360°, is called the constant of the cone (usually denoted by n). 'n' is always less than unity. The Pole nearer the apex becomes the centre of the circle of the sector and the meridians become the radii. The angle between any two meridians on the map is 'n' times the angle between the corresponding meridional planes on the globe. The latitude circles become arcs of the sector spaced in such a way as to make the map conformal. The polar coordinate system is convenient in discussing Lambert's Conformal Conical and Polar Stereographic Projections.

7.2 The meridian along which the cone is cut open is the bounding meridian which becomes the bounding radii of the sector. If the longitude λ on the earth's surface is measured from this meridian and if, on the map, the longitude angle θ is measured from the corresponding bounding radius (Fig. 8b),

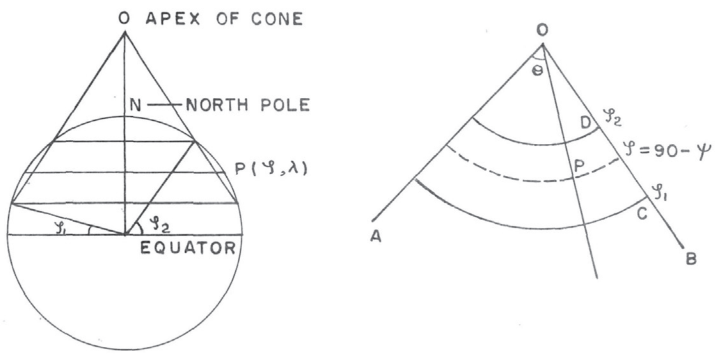

FIG. 8(a) FIG. 8(b)

FIG. 8: LAMBERT'S CONFORMAL CONICAL PROJECTION

Cone cutting sphere at standard parallels φ_1 and φ_2.

Graticule of Lambert's Conformal Conical Projection.

O , Projection of north Pole on the map.

OA and OB are bounding radii (meridians).

$OP = r$, $OC = r_1$, $OD = r_2$

$\angle AOB = 2\pi n$.

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* See foot note on page 1.

$$\theta = n\lambda ,$$

and $$\frac{d\theta}{d\lambda} = n \quad \ldots(7.1)$$

The differential equation for conformal projection in the case of polar co-ordinates (vide Appendix 2) is,

$$\frac{dr}{d\psi} = r \operatorname{cosec}\psi \frac{d\theta}{d\lambda}$$

$$= n\, r \operatorname{cosec}\psi .$$

Therefore,

$$\frac{dr}{r} = n \frac{d\psi}{\sin\psi} .$$

Integrating

$$\log r = \log \left(\tan\frac{\psi}{2}\right)^n + C_1 ,$$

hence

$$r = C_2 \left(\tan\frac{\psi}{2}\right)^n .$$

Since the scale is true along latitude φ_1 and φ_2 (or colatitudes ψ_1 and ψ_2), comparing the lengths of the standard parallel (φ_1) on the earth and its length on the projection, we get

$$2\pi n r_1 = 2\pi a \sin\psi_1, \text{ or}$$

$$r_1 = \frac{a}{n} \sin\psi_1 .$$

Similarly,

$$2\pi n r_2 = 2\pi a \sin\psi_2, \text{ or}$$

$$r_2 = \frac{a}{n} \sin\psi_2 .$$

Using the above equations to eliminate C_2,

$$r = \frac{a}{n} \sin\psi_1 \left\{\frac{\tan\frac{\psi}{2}}{\tan\frac{\psi_1}{2}}\right\}^n$$

$$= \frac{a}{n} \sin\psi_2 \left\{\frac{\tan\frac{\psi}{2}}{\tan\frac{\psi_2}{2}}\right\}^n . \quad \ldots(7.2)$$

Thus two equations, in terms of either ψ_1 or ψ_2, are obtained either of which can be used.

Also,

$$\frac{r_1}{r_2} = \frac{\sin\psi_1}{\sin\psi_2}$$

$$= \left\{\frac{\tan\frac{\psi_1}{2}}{\tan\frac{\psi_2}{2}}\right\}^n .$$

Taking logarithms of either side of the above equation,

$$n\left(\log\tan\frac{\psi_1}{2} - \log\tan\frac{\psi_2}{2}\right) = \log\sin\psi_1 - \log\sin\psi_2 .$$

Therefore,

$$n = \frac{\log\sin\psi_1 - \log\sin\psi_2}{\log\tan\frac{\psi_1}{2} - \log\tan\frac{\psi_2}{2}} . \quad \ldots(7.3)$$

This equation defines the relationship between n, ψ_1, and ψ_2; in other words n is uniquely determined by our choice of ψ_1 and ψ_2.

From this, we get

n = 0.716 for Standard Parallels 30° and 60°N

= 0.428 ,, ,, 10° and 40°N .

Using the appropriate value of n, the angle of the sector of the projectic and also the distance between the consecutive meridians can be calculated. Then the sector and the meridians can be drawn. Computing the values of r, at suitable intervals, by the use of eq. (7.2), the latitude arcs can be drawn. This done, the graticule is ready for further use. A world map on Lambert's Conformal Conical Projection is shown in Fig. 9 (page 10).

7.3 The scale m for this projection is given by

$$m = \frac{2\pi n r}{2\pi a \sin\psi}$$

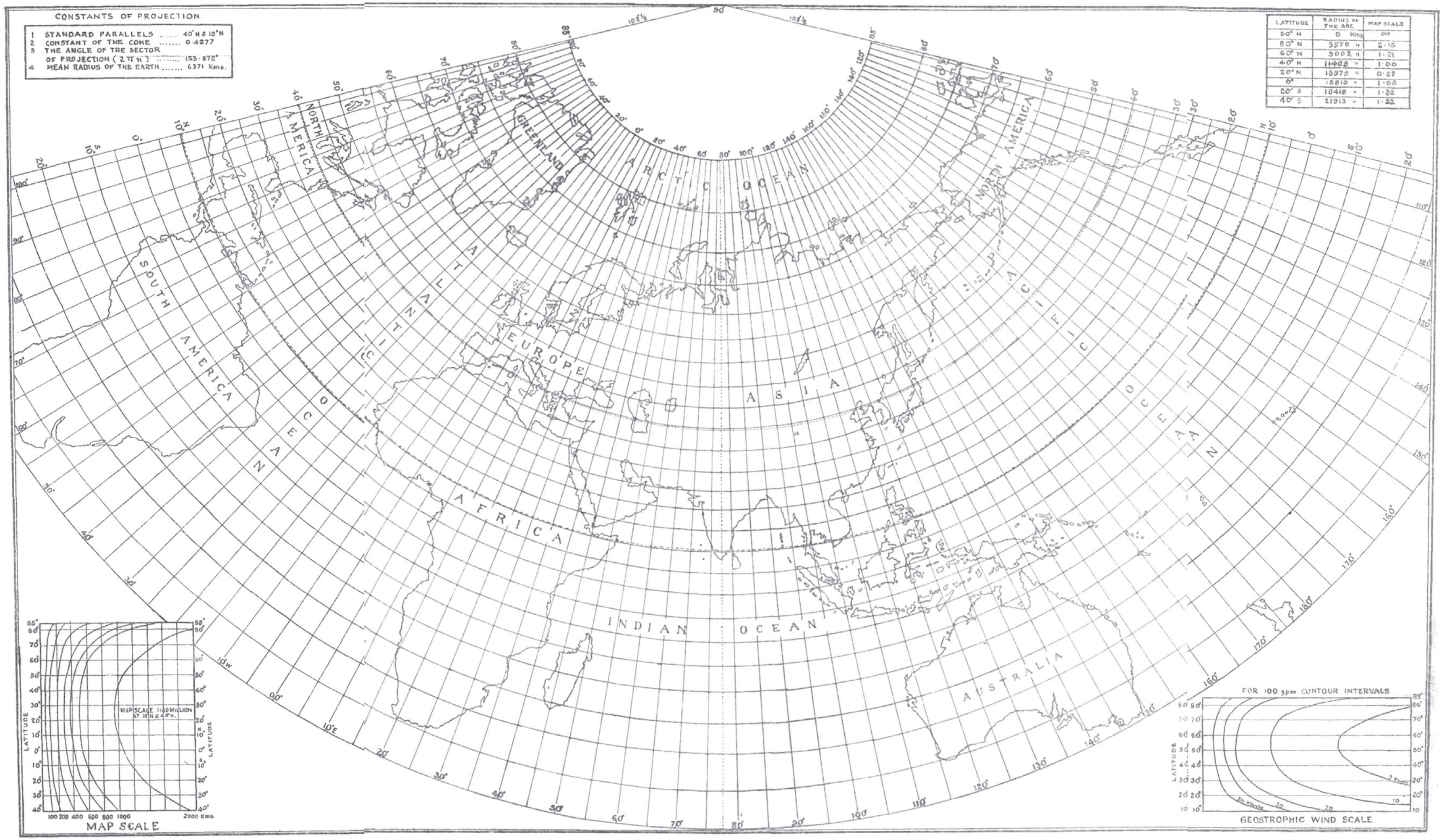
$$= \frac{\sin\psi_1}{\sin\psi} \left\{\frac{\tan\frac{\psi}{2}}{\tan\frac{\psi_1}{2}}\right\}^n ,$$

or

$$m = \frac{\sin\psi_2}{\sin\psi} \left\{\frac{\tan\frac{\psi}{2}}{\tan\frac{\psi_2}{2}}\right\}^n \quad \ldots(7.4)$$

after substituting for r from eq. (7.2).

Either of the Equations will suffice. It is evident from the above equations that the scale is equal to unity when ψ equals ψ_1 or ψ_2. As we



WORLD MAP

LAMBERT'S CONFORMAL CONICAL PROJECTION WITH SCALE TRUE AT 40°N & 10°N.

FIG. 9.

proceed away from both these latitudes towards the Poles the value of the scale-ratio increases until at last it is infinite at the Poles. The scale at the Equator is 1.28 if the Standard Parallels are 30°N and 60°N and 1.06 if they are 10°N and 40°N. Between the two parallels, the scale is less than unity, reaching a minimum value of 0.97 in between. If the cone touches the earth along colatitude ψ_o,(Fig. 10) the radius of the arc representing this latitude circle is given by

$$r_o = a \tan \psi_o .$$

Also $2\pi n r_o = 2\pi a \sin \psi_o$; for the scale is true along this latitude.
Therefore

$$n = \cos \psi_o . \qquad \ldots(7.5)$$

After substituting the appropriate values in eqs. (7.2) and (7.4)

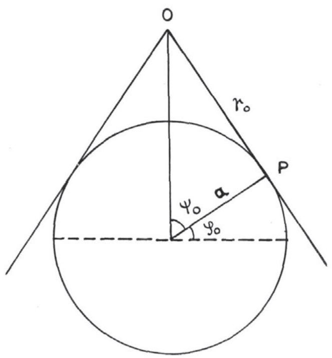
$$r = a \tan \psi_o \left\{ \frac{\tan \frac{\psi}{2}}{\tan \frac{\psi_o}{2}} \right\}^{\cos \psi_o} \qquad \ldots(7.6)$$

and

$$m = \frac{nr}{a \sin \psi}$$

$$= \frac{\sin \psi_o}{\sin \psi} \left\{ \frac{\tan \frac{\psi}{2}}{\tan \frac{\psi_o}{2}} \right\}^{\cos \psi_o} \qquad \ldots(7.7)$$

The scale is unity along the Standard Parallel $(90 - \psi_o)$ and increases away from it.



The Cone touches the earth at P.

FIG. 10 : LAMBERT'S CONFORMAL CONICAL PROJECTION

7.4 In this connection it may be of interest to note that the scale at the Pole is infinite in both Mercator's and Lambert Conformal Conical Projections. However, the Pole can be represented only in the Lambert's Conformal Conical Projection while it cannot be represented in the Mercator's Projection. The Polar Stereographic Projection is different from the other two projections, in the respect, that the Pole can be represented on this Projection and the scale at the Pole has also a finite value.

7.5 The merit of the Lambert's Conformal Conical Projection with two standard Parallels, lies in the fact that by choosing the appropriate Standard parparallel the distortion over the area or region for which the map is used can be reduced to a minimum. The two latitudes may be so chosen that they divide the region concerned into three nearly equal parts. This projection is not evidently suitable for the polar regions since the scale is infinite at the Poles. It is particularly suitable for the middle latitudes. It is not quite unsuitable to include the equatorial areas in addition to the middle latitudes, provided that lower parallels are chosen as standard. Given the maximum permissible scale of magnification there is a limit to the southernmost latitude which can be covered on the map. Appendix 6 provides the necessary information.

8. The Polar Stereographic Projection

8.1 In the case of Polar and Circumpolar Stereographic Projections (Fig. 11) the Pole is at the centre of the map. The meridians are straight lines radiating from the centre and inclined to each other at the same angle as the corresponding meridional planes. The latitudes are concentric circles centred at the Pole and placed at such a distance as to make the scale at any point the same along the meridian and the latitude.

If r, θ are the polar coordinates of any point 'P' on the map,

$$\theta = \lambda \qquad \ldots(8.1)$$

and hence

$$\frac{d\theta}{d\lambda} = 1 .$$

The differential equation for conformality (vide Appendix 2) is

$$\frac{dr}{d\psi} = r \operatorname{cosec} \psi \frac{d\theta}{d\lambda}$$

$$= \frac{r}{\sin \psi} .$$

Therefore

$$\frac{dr}{r} = \frac{d\psi}{\sin \psi} .$$

Integrating,

$$r = c \tan \frac{\psi}{2} \quad . \qquad \ldots(8.2)$$

If the scale is true along colatitude ψ_o whose radius on the map is r_o, then

$$r_o = c \tan \frac{\psi_o}{2} \quad .$$

It will be seen from Fig. 11b

$$r_o = FG = FS \tan \frac{\psi_o}{2}$$

$$= (FO + OS) \tan \frac{\psi_o}{2}$$

$$= a(1 + \cos \psi_o) \tan \frac{\psi_o}{2} \quad .$$

Comparing these two expressions for r_o,

$$c = a(1 + \cos \psi_o) \; .$$

Substituting for c in eq. 8.2,

$$r = a(1 + \cos \psi_o) \tan \frac{\psi}{2}$$

$$= 2a \cos^2 \frac{\psi_o}{2} \tan \frac{\psi}{2} \quad . \qquad \ldots(8.3)$$

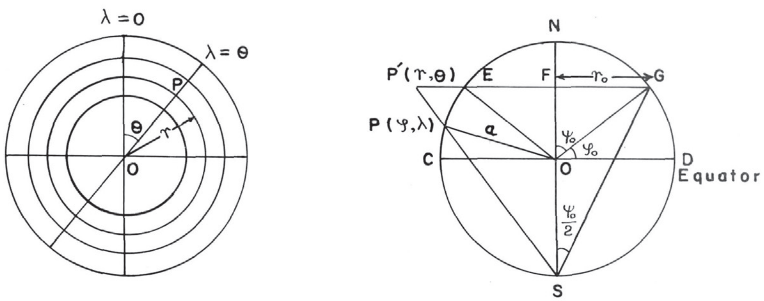


FIG. 11(a)   FIG. 11(b)

FIG. 11 : POLAR STEREOGRAPHIC PROJECTION

Graticule of Polar Stereographic Projection.

Vertical cross-section of plane cutting a sphere at standard parallel φ_o.

8.2 The scale m of this projection is,

$$m = \frac{2\pi r}{2\pi a \sin \psi} \qquad \ldots(8.4)$$

$$= \frac{1 + \cos \psi_o}{\sin \psi} \tan \frac{\psi}{2}$$

$$= \frac{1 + \cos \psi_o}{2 \cos^2 \frac{\psi}{2}}$$

$$= \frac{\cos^2 \frac{\psi_o}{2}}{\cos^2 \frac{\psi}{2}}$$

If the plane of projection touches the sphere at the Pole, $\psi_o = 0$ and

and

$$r = 2a \tan \frac{\psi}{2} \qquad \ldots(8.5)$$

$$m = \sec^2 \frac{\psi}{2} \qquad \ldots(8.6)$$

The scale is unity at the Standard Parallel and increases with decreasing latitude, becoming infinite at the South Pole. It decreases towards North Pole at which the minimum value is attained. In the case of the projection with Standard Parallel at 60°N the scale varies from 0.93 to 1.86 between the Pole and the Equator.

8.3 The Polar Stereographic Projection has certain merits besides being the most appropriate projection for the Polar regions. It can easily be constructed. The areas represented are continuous without any break anywhere and the map is symmetric round any meridian. The Mercator's and Lambert's projections split the continents lying at the ends of the map and can be made symmetric only around any chosen meridian. A complete hemisphere including the Pole can be represented on the Polar Stereographic Projection unlike in the case of say, the Mercator's Projection. The whole surface of the earth can also be represented by placing Polar Stereographic maps of the northern and southern hemisphere side by side. Fig. 12 illustrates the map of northern hemisphere in this projection.

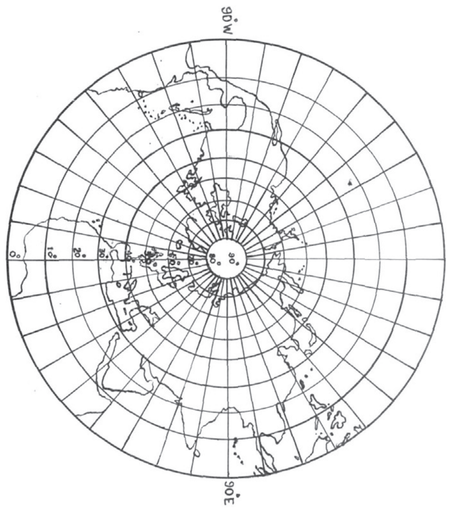


FIG. 12 : THE POLAR STEREOGRAPHIC PROJECTION (Hemisphere Map)

[From 'An Introduction to the Study of Map Projections' by J.A. Steers 1965 (Fourteenth Edition)]

9. Representation of Scale of Maps

9.1 As will be seen from what has been stated in the above paragraphs, in any projection, the scale of the map is true only at the Standard Parallel; at the other latitudes, the scale gets modified on account of the inevitable distortion arising out of the projection. Thus at any parallel other than the Standard Parallel, a correction (called map factor*) has to be applied to map distances. Another point to be taken into account is that the vast area of the globe has to be reduced by a suitable ratio such as $1:10^6$ or $1:10^7$, (called the Representative Fraction 's) before they could be represented on maps.

9.2 When we estimate distances between places from a map, it is therefore necessary to take into account both the Representative Fraction,'s' and the Map Factor, 'm'. The scale will be equal to 's' only along the Standard Parallels where m = 1. If the area covered is so small, that is, if m does not change significantly, the scale can be denoted by 's' alone. In cases where one figure can represent the scale for the whole map, it can be represented as simply a fraction or as "1 inch represents so many miles" or "1 cm represents so many kms". It can also be represented by means of a graduated scale by drawing a line and dividing it at map distances that represent real distances say 100 km, 200 km and so on. This will enable one to measure the map distance by dividers and get the true distance by measuring the divider length on the scale. In the case of large maps where the scale changes from latitude to latitude an echelon of such graduated lines is given. In such an echelon if the lines corresponding to different latitudes are placed at distances proportionate to the latitudes, the points representing a given distance will, in general, lie on a curve. But spacing of the latitude lines can be suitably adjusted to make the curve a straight line. The disadvantage of this method is that intermediate latitudes do not fall at proportionate distances; for example, the latitude 35° does not lie half way between 30° and 40°. Its position cannot be interpolated easily. Fig. 13 illustrates different ways of representing the scale of the map. The choice of the scale of any map will naturally depend on the network of observations to be plotted on it, so that the scale is neither too small to create confusion between adjacent observations nor too large to leave wide gaps between plotted observations.

* The terms 'Map Factor' and 'Map Scale' are used synonymously.

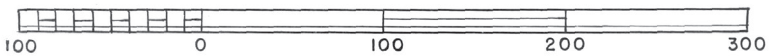


FIG. 13 (i) : SCALE AT A SINGLE LATITUDE

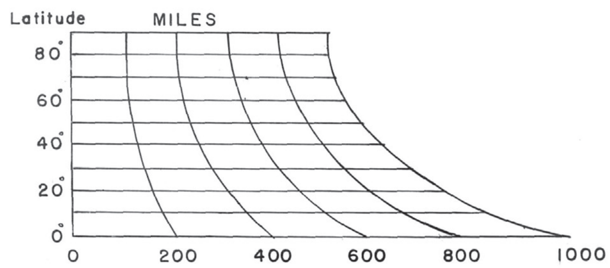


FIG. 13 (ii) (a) : VARIATION OF SCALE WITH LATITUDE

POLAR STEREOGRAPHIC PROJECTION

SCALE – $1:27 \times 10^6$ AT LATITUDE 60°N

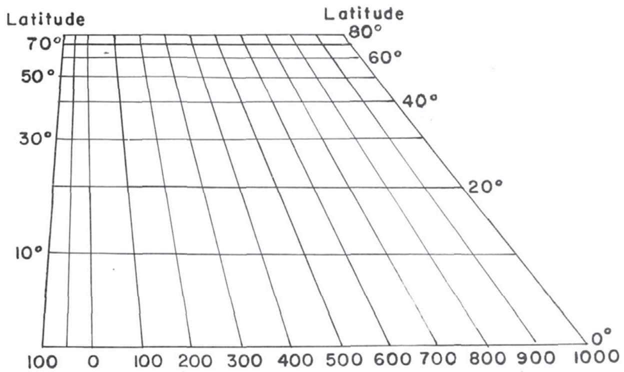


FIG. 13 (ii) (b): POLAR STEREOGRAPHIC PROJECTION

SCALE:-(TRUE AT LATITUDE 60°N) 1:27X10^6

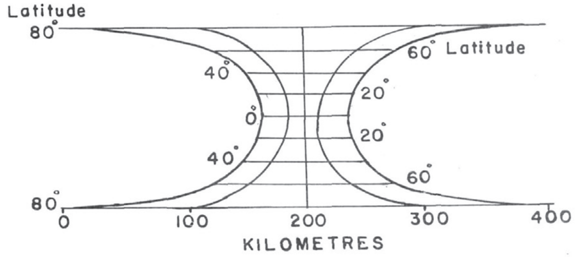


FIG. 13 (ii) (c): MERCATOR'S PROJECTION

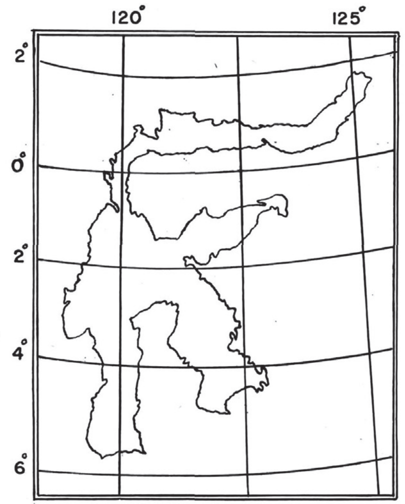
SCALE:- 1:2 X 10^6 AT 22°30' LATITUDE

FIG. 13: METHODS OF REPRESENTING SCALES OF PROJECTION

9.3 To find the distance of a line on the map, divide the line into suitable segments, measure the distance of each segment against the scale corresponding to the mean latitude of the segment and add up these distances. The distance can also be measured by determining the number of degrees of latitude to which each of the segment corresponds. The distances in degrees can be converted into miles or kilometers, since the number of miles or kilometers in one degree of latitude along meridian is known exactly.

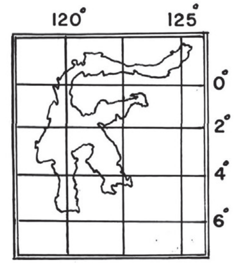
9.4 The relative sizes of geographical features may appear different in different projections, even with the same scale of reduction, although shapes are preserved. This is due to the differences in the variation of map factor with latitude in each of the projections. With the same Representative Fraction, Greenland in Mercator's Projection appears four times as large as in Stereo-

look only one half as small as in Polar Stereographic Projection.



Polar Stereographic Projection

Scale: 1:2 x 10^7 at 60°N



Mercator's Projection

Scale: 1:2 x 10^7 at 22°.30' latitude

FIG. 14: CELEBES IN TWO DIFFERENT PROJECTIONS

Note: Scale is the same along respective standard latitudes.

10. Great Circles on a Map

10.1 Another matter of interest in map projection is to know how the Great Circles transform. A Great Circle is the shortest distance along the surface of a sphere between two points on it. The Great Circle arc joining two points can be mapped with the help of the Gnomonic Projection. Earlier it was seen (vide Sec 3) that, in the Gnomonic Projection, the Great Circle arc becomes a straight line. Take a Gnomonic graticule, plot the two points on it, join them by a straight line, determine the coordinates of a sufficient number of points on the line and plot these on the required projection, you get the great circle distance between the two points. Figs. 2 and 8 show the delineation of Great Circles on the Gnomonic and the Mercator's Projections. In the Mercator's Projection all Great Circles, other than the Equator and meridians are curved convex towards the nearer Pole, the curvature increasing with latitude. On the Polar Stereographic Projection, Great Circles other than the meridians appear as lines curved concave towards the Pole.

11. Projections and Meteorological Analysis

mical parameters used in meteorological analysis. The actual distance between two places is required in the calculation of the geostrophic wind. This distance can be calculated from the map distance by dividing it by the suitable scale factor. In order to facilitate computation, a formula can be obtained using the map distance and the scale, so that the geostrophic wind at any latitude can be reckoned directly from the map distance. A scale can be prepared for the calculation of the geostrophic wind taking into account the scale variation. The computation of vorticity from a chart is also affected by the projection used. Appendix 5 discusses how the geostrophic wind and geostrophic vorticity can be computed from the chart. The horizontal component of the curvature of a trajectory which enters the calculations of gradient wind is also affected by the projection used, since the curve on the sphere is modified to some extent in projection. Appendix 4 discusses this point in some detail.

11.2 As will be seen from the Table given in Appendix 6, in the case of Mercator's Projection with standard parallel $22\frac{1}{2}^{\circ}$N and $22\frac{1}{2}^{\circ}$S, the maximum distortion is less than 8% for the area lying between 30°N and 30°S. In the case of Lambert's Conformal Conical Projection with Standard Parallels 10°N and 40°N, the maximum distortion is less than 7% for the area lying between the Equator and 50°N. In the case of Polar Stereographic Projection the maximum distortion is less than 9% for the area north of 45°N. Thus over large areas of the map the error due to map distortion is generally negligible compared to the errors of observation and analysis.

11.3 On the weather maps the wind directions are to be plotted with reference to true north. Except in the case of Mercator's Projection, since orientation of the meridians are different on different projections, as well as on different sections of the same map, it is necessary to note the direction of true north at each point while plotting the wind observations.

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APPENDICES

Besides Appendices 3, 4, and 5 which have been referred to above, three more appendices have been added to the text. Appendix 1 gives the essentials of Conformal Projections. Appendix 2 gives the Mathematical Theory of Conformal Projections. Appendix 6 contains a table showing the variations of scale at different latitudes in the three Conformal Projections discussed above.

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APPENDIX – 1

The Essentials of Conformal Projections.

Feature \ Projection	Mercator's	Lambert's Conformal Conical	Polar Stereographic
1. Projection of meridians	Vertical lines	Radii of a sector of circles	Radii of circles
2. Projection of latitude circles	Horizontal lines	Arcs of concentric circles	Full concentric circles
3. Recommended* Standard Parallels (φ_o or φ_1 and φ_2)	φ_o (22.5°N and S)	φ_1 and φ_2 (30°N and 60°N or 10°N and 40°N)	φ_o (60°N)
4. Distance of parallels from the central line or from the centre ($\psi = 90 - \varphi$)	$a \cos \varphi_o \log \tan \left(\frac{\pi}{4} + \frac{\varphi}{2}\right)$	$\frac{a}{n} \sin \psi_1 \left\{\frac{\tan \psi/2}{\tan \psi_1/2}\right\}^n$ or $\frac{a}{n} \sin \psi_2 \left\{\frac{\tan \psi/2}{\tan \psi_2/2}\right\}^n$ where $n = \frac{\log \sin \psi_1 - \log \sin \psi_2}{\log (\tan \psi_1/2) - \log (\tan \psi_2/2)}$	$a\,(1 + \cos \psi_o) \tan \psi/2$
5. Distance or angular distance between meridians 0° and λ°	$\lambda\, a \cos \varphi_o$	$n\,\lambda$ degrees	λ degrees
6. m, the scale at any latitude	$\cos \varphi_o / \cos \varphi$	$\frac{\sin \psi_1}{\sin \psi} \left\{\frac{\tan \psi/2}{\tan \psi_1/2}\right\}^n$ or $\frac{\sin \psi_2}{\sin \psi} \left\{\frac{\tan \psi/2}{\tan \psi_2/2}\right\}^n$	$\frac{\cos^2 \psi_o/2}{\cos^2 \psi/2}$
7. Scale value at selected latitudes	Equator 0.92; 10°N and S 0.94; 22.5°N and S 1.00; 40°N and S 1.21 60°N and S 1.85; 75°N and S 3.57; 90°N and S ∞	(Standard Parallels 10°N and 40°N) 90°N ∞; 75°N 1.72; 60°N 1.21; 40°N and 10°N 1.00 25°N 0.97; Equator 1.06; 10°S 1.16	(Standard Parallel 60°N) 90°N 0.93; 60°N 1.00; 30°N 1.24; 0° 1.86
8. Latitudes between which the scale distortion is less than	5%: 28°N and 28°S; 10%: 32°N and 32°S 15%: 36°N and 36°S; 20%: 39°N and 39°S	5%: 48°N to 2°N; 10%: 53°N to 4°S 15%: 57°N to 8°S; 20%: 61°N to 14°S (m = 1 at 10° and 40°N)	5%: 50°N; 10%: 43°N; 15%: 38°N; 20%: 37°N
9. Geostrophic wind (V_g) and Geostrophic vorticity(q)	$V_g = mV'_g \qquad q = m^2 q'$ (the prime denotes the values measured from the map)		

* See Appendix 3.

APPENDIX – 2.

Mathematical Theory of Conformal Projection

Symbols used

a	–	radius of the earth.
φ and λ	–	latitude and longitude of Q, any point on the earth's surface.
x, y	–	Coordinates of Q in Rectangular Cartesian Coordinate System.
r, θ	–	Coordinates of Q in Polar Coordinate System.
dS	–	Small distance element on the surface of the earth.
ds	–	Corresponding small distance element on the map.

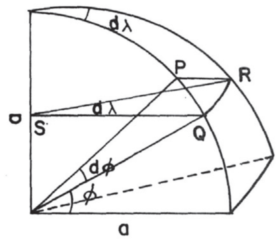

FIG. 15

$PQ = a\,d\varphi$

$SQ = a\cos\varphi$

$QR = a\cos\varphi\,d\lambda$

$PR^2 = PQ^2 + QR^2$

In this discussion, the earth is considered as a perfect sphere. From Fig. 15 it will be seen

$$dS^2 = a^2\,(d\varphi)^2 + a^2\cos^2\varphi\,(d\lambda)^2 .$$

In the Cartesian Coordinate system,

$$ds^2 = dx^2 + dy^2 .$$

Now,

$$dx = \frac{\partial x}{\partial \lambda}\,d\lambda + \frac{\partial x}{\partial \varphi}\,d\varphi \quad \text{and}$$

$$dy = \frac{\partial y}{\partial \lambda}\,d\lambda + \frac{\partial y}{\partial \varphi}\,d\varphi .$$

Therefore,

$$ds^2 = \left\{\left(\frac{\partial x}{\partial\lambda}\right)^2 + \left(\frac{\partial y}{\partial\lambda}\right)^2\right\}(d\lambda)^2 + \left\{\left(\frac{\partial x}{\partial\varphi}\right)^2 + \left(\frac{\partial y}{\partial\varphi}\right)^2\right\}(d\varphi)^2$$
$$+ 2\left\{\frac{\partial x}{\partial\lambda}\,\frac{\partial x}{\partial\varphi} + \frac{\partial y}{\partial\lambda}\,\frac{\partial y}{\partial\varphi}\right\}d\lambda\,d\varphi .$$

According to the definition of a conformal projection, at any point (φ, λ) the scale $\frac{ds}{dS}$ is same in all directions (i.e.) it is independent of $\frac{d\lambda}{d\varphi}$, since $\frac{ds}{dS}$ represents the scale at that point which is a constant irrespective of the orientation of the distance element dS.

Therefore

$$\frac{ds^2}{dS^2} = \text{Constant, } k;$$

that is

$$ds^2 = k\,dS^2 .$$

Therefore,

$$\left\{\left(\frac{\partial x}{\partial\lambda}\right)^2 + \left(\frac{\partial y}{\partial\lambda}\right)^2 - ka^2\cos^2\varphi\right\}(d\lambda)^2 + \left\{\left(\frac{\partial x}{\partial\varphi}\right)^2 + \left(\frac{\partial y}{\partial\varphi}\right)^2 - ka^2\right\}(d\varphi)^2$$
$$+ 2\left\{\frac{\partial x}{\partial\lambda}\,\frac{\partial x}{\partial\varphi} + \frac{\partial y}{\partial\lambda}\,\frac{\partial y}{\partial\varphi}\right\}d\lambda\,d\varphi = 0$$

This equation is true for all values of $d\lambda$ and $d\varphi$; and therefore, the coefficients of the equation in $d\lambda$ and $d\varphi$ should be zero.

That is,

$$\left(\frac{\partial x}{\partial\lambda}\right)^2 + \left(\frac{\partial y}{\partial\lambda}\right)^2 = ka^2\cos^2\varphi$$

$$\left(\frac{\partial x}{\partial\varphi}\right)^2 + \left(\frac{\partial y}{\partial\varphi}\right)^2 = ka^2$$

and

$$\frac{\partial x}{\partial\lambda}\,\frac{\partial x}{\partial\varphi} + \frac{\partial y}{\partial\lambda}\,\frac{\partial y}{\partial\varphi} = 0 .$$

From these equations, we get

$$\frac{\frac{\partial x}{\partial\lambda}}{\frac{\partial y}{\partial\lambda}} = -\frac{\frac{\partial y}{\partial\varphi}}{\frac{\partial x}{\partial\varphi}} = c\text{, a constant,}$$

$$\left(\frac{\partial y}{\partial\lambda}\right)^2 (1 + c^2) = ka^2\cos^2\varphi \quad \text{and}$$

$$\left(\frac{\partial x}{\partial \varphi}\right)^2 (1 + c^2) = ka^2 .$$

Eliminating k and c,

$$\left(\frac{\partial y}{\partial \lambda}\right)^2 = \left(\frac{\partial x}{\partial \varphi}\right)^2 \cos^2 \varphi$$

or

$$\frac{\partial y}{\partial \lambda} = \pm \frac{\partial x}{\partial \varphi} \cos \varphi . \qquad \ldots 1(a)$$

Similarly

$$\frac{\partial x}{\partial \lambda} = \pm \frac{\partial y}{\partial \varphi} \cos \varphi . \qquad \ldots 1(b)$$

Similarly in Polar Coordinates, where $ds^2 = (dr)^2 + r^2 (d\theta)^2$,

$$\frac{\partial r}{\partial \lambda} = \pm r \frac{\partial \theta}{\partial \varphi} \cos \varphi \qquad \ldots 2(a)$$

and

$$\frac{\partial \theta}{\partial \lambda} = \pm \frac{1}{r} \frac{\partial r}{\partial \varphi} \cos \varphi . \qquad \ldots 2(b)$$

If the axis of the image surface (cylinder or cone) is chosen to coincide with the axis of the earth and x or θ to increase with λ ,

$$\frac{\partial y}{\partial \lambda} = \frac{\partial x}{\partial \varphi} = 0 . \qquad \ldots (3)$$

$$\frac{\partial r}{\partial \lambda} = \frac{\partial \theta}{\partial \varphi} = 0 . \qquad \ldots (4)$$

Algebrically, east longitudes and north latitudes are considered positive and west longitudes and south latitudes, negative. Accordingly, the longitudes increase with x in the Cartesian and with θ in the Polar co-ordinate systems. This makes $\frac{\partial x}{\partial \lambda}$ and $\frac{\partial \theta}{\partial \lambda}$ both positive. On the other hand, the latitude increases with y in the former but decreases with r (where the Pole is the origin of r) in the latter system. This makes $\frac{\partial y}{\partial \varphi}$ positive and $\frac{\partial r}{\partial \varphi}$ negative.

Hence choosing the relevant signs from eq. 1 and 2

$$\frac{\partial y}{\partial \varphi} = \sec \varphi \frac{\partial x}{\partial \lambda} \text{ , from eq. (1b)} \qquad \ldots (5)$$

$$\frac{\partial r}{\partial \varphi} = - r \sec \varphi \frac{\partial \theta}{\partial \lambda} \text{ , from eq. (2b)} \qquad \ldots (6)$$

If we substitute φ by $(90 - \psi)$, equations (4) and (6) become

$$\frac{\partial r}{\partial \lambda} = \frac{\partial \theta}{\partial \psi} = 0 \qquad \ldots (7)$$

and

$$\frac{\partial r}{\partial \psi} = \frac{r}{\sin \psi} \frac{\partial \theta}{\partial \lambda} \qquad \ldots (8)$$

Eq. (3) and (5) are the differential equations for the Mercator's Projection and eq. (7) and (8) are for the Lambert's Conformal Conical or Polar Stereographic Projections.

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APPENDIX - 3.

Extracts from Chapter 1 of WMO. No. 151, TP. 71 "Guide to the preparation of Synoptic Weather Charts and Diagrams".

1.1.1 Standard projection of base maps.

The following projections should be used for weather charts:

(a) The Stereographic Projection for the polar region on a plane cutting the sphere at the Standard Parallel of latitude 60°.

(b) Lambert's Conformal Conic Projection for middle latitudes, the cone cutting the sphere at the Standard Parallels of latitude 10° and 40° or 30° and 60°.

(c) Mercator's Projection for the equatorial regions, with true scale at the Standard Parallel of latitude 22.5°.

1.1.2 Standard Scale of base maps.

The scale along the standard parallel should be as follows for weather charts:

(a)	Covering the world	1 : 40,000,000
(b)	Covering the hemisphere	1 : 30,000,000
(c)	Covering a large part of a hemisphere	1 : 20,000,000
(d)	Covering a continent or an ocean or considerable parts of either or both	1 : 7,500,000 or
		1 : 10,000,000 or
		1 : 12,500,000 or
		1 : 15,000,000

1.1.3 The name of the projection, the scale at the Standard Parallels and the scales for other latitudes should be indicated on every weather chart.

APPENDIX - 4.

The Effect of Polar Stereographic Projection on the Calculation of the Curvature of Horizontal Curves

The Polar Stereographic Projection is used for several purposes especially for weather charts in the middle and higher latitudes. Sometimes it is of interest to compute the curvature of certain contours as well as other types of curves like isobars from the curvature of their projections on this chart. The relationship (Krishna 1968) between the curvatures (K_H and K'_H) of a horizontal curve and its projection on a Polar Stereographic Chart is given by,

$$K_H = m\left\{ K'_H - \frac{\sin\psi \cos\beta}{a\,(1+\cos\psi)} \right\} \qquad \ldots (1)$$

where

ψ = the colatitude $(90 - \varphi)$,

β = angular distance between the latitude circle and the curve whose curvature is K_H,

a = radius of the earth,

and m = map factor.

Eq. (1) gives the relation between K_H and K'_H for a Stereographic Projection from the South Pole on a plane cutting the sphere at any latitude. As particular cases

$$K_H = m\left\{ K'_H - \frac{\sin\psi \cos\beta}{2a} \right\} \qquad \ldots (2)$$

where the plane of projection touches the sphere at the Pole and

$$K_H = m\left\{ K'_H - \frac{\sin\psi \cos\beta}{a} \right\} \qquad \ldots (3)$$

where the plane of projection is the Equator.
Equation (1) can be simplified to

$$K_H = mK'_H - \frac{1}{a}\tan\frac{\psi}{2}\cdot\cos\beta\,. \qquad \ldots (4)$$

Following two limiting cases are noteworthy:-

(a) in the case of meridian circles,

$\beta = 90$, $\cos\beta$ and K_H equal 0 and therefore $K'_H = 0$ indicating that they project into straight lines. This is obvious, as in Polar Stereographic Projection the meridians project into radial lines, cutting the latitude circles at right angles.

(b) in the case of latitude circles

$$\beta = 0$$

and

$$K_H = mK'_H - \frac{\tan\frac{\psi}{2}}{a}\,.$$

Haltiner and Martin (1957) in their book, on 'Dynamical and Physical Meteorology' give only equation (2). Equation (1) is more general and can be applied for Circumpolar Stereographic Projection with Standard Parallel 60°N, recommended by the WMO.

The difference between K_H and mK'_H is given by $\frac{1}{a}\tan\psi/2\ \cos\beta$. It vanishes where $\beta = 90°$, i.e. where the curve touches the meridian. If the curvature is measured at this point, K_H is simply equal to mK'_H where m is the scale at the latitude of the point of contact. If the trajectory is a circle, the curvature is the same all along the circle and so K_H can be computed from K'_H by taking the scale at the point of contact of either of the meridians touching the circle.

The maximum (minimum) difference between K_H and mK'_H occurs where β is 0° (or 180°) i.e. when $\cos\beta = \pm 1$, where the meridian cuts the curve orthogonally. Here difference is $\tan\psi/2$ times the earth's curvature. In other words, in the Northern Hemisphere the difference is always less than the earth's curvature. In the Southern Hemisphere, however, it can exceed unity.

An interesting effect of the projection is that, whereas a circle projects into a circle, its centre does not project into the centre of its projection. The centre of the projection of a circle lies farther away from the Pole than the projection of the centre of the circle.

If the centre of the circle lies on the Equator the apparent shift of the centre of projection of a circle is approximately 15' and 1° for circles of radius 5° and 10° respectively. The corresponding values for centres at 30°N lat. are 9' and 35' respectively.

Since the centre of the projected circle is not the same as the projection of the centre of the small circle, concentric small circles other than the latitude circles will not project into concentric circles. The centres of projection of these concentric circles will be different but will lie on the same meridian;

the centre of a circle with a larger radius will be displaced farther away from the Pole.

The complications introduced by the distortions due to the projection can be avoided if the diameter (2b) of the circle of curvature is taken as the difference between the latitudes of the points where the meridian through the centre intersects the circle of curvature and K_H is calculated from its definition ($K_H = \frac{1}{a} \cot b$).

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APPENDIX - 5.

Computation of Dynamical Parameters from Conformal Maps

A. Geostrophic Wind

$$V_g = \frac{g}{f} \frac{\Delta Z}{\Delta H}$$ is the equation for geostrophic wind,

where g is the acceleration due to gravity,

f , the coriolis parameter,

Δ Z, the contour difference and

Δ H, the actual distance between the contours.

But

$$\Delta h = ms \Delta H$$

where

Δ h is the distance between contours on a conformal map,

m, the map factor,

s, the representative fraction.

Therefore

$$V_g = \frac{g}{2\omega \sin \varphi} \frac{sm \Delta Z}{\Delta h}$$

$$= \frac{gs \Delta Z}{2\omega} \frac{m}{\sin \varphi} \times \frac{1}{\Delta h}$$

$$= m V'_g$$

where

V'_g is the wind calculated from the map without allowing for change of scale.

Since the quantity $\frac{gs \Delta Z}{2\omega}$ is a constant and $\frac{m}{\sin \varphi}$ depends upon latitude only, nomograms can be drawn with different values of φ for reading off V_g from Δ h.

In the Mercator's Projection

$$\frac{m}{\sin \varphi} = \frac{\cos \varphi_o}{\cos \varphi \sin \varphi}$$

$$= \frac{2 \cos \varphi_o}{\sin 2 \varphi} .$$

In the Conic Projection

$$\frac{m}{\sin \varphi} = \frac{\sin \psi_1}{\sin \psi \cos \psi} \left\{ \frac{\tan \psi/2}{\tan \psi_1/2} \right\}^n$$, where ψ is the colatitude

$$= \frac{2 \sin \psi_1}{\sin 2 \psi} \left\{ \frac{\tan \psi/2}{\tan \psi_1/2} \right\}^n .$$

In the Polar Stereographic Projection

$$\frac{m}{\sin \varphi} = \frac{1 + \sin \varphi_o}{(1 + \sin \varphi) \sin \varphi} .$$

The values of $m/\sin\varphi$ at selected latitudes are given below for each of the three conformal projections.

Table

Latitude °N	Mercator's Projection m = 1 at $22\frac{1}{2}$°	Lambert's conformal m = 1 at 60° and 30°N	Polar Stereographic m = 1 at 60°N
80	5.403	1.313	0.954
70	2.875	1.154	1.024
60	2.134	1.155	1.155
50	1.876	1.264	1.380
40	1.876	1.509	1.767
30	2.134	2.000	2.488
20	2.875	3.093	4.064

B. Geostrophic Vorticity

$q = 1/A \oint V_{gs}\, ds$ is the equation for geostrophic vorticity, where

q is the geostrophic vorticity,

A, the area of an infinitesimal closed curve,

V_{gs}, the component of geostrophic wind along the boundary of the curve.

$q' = 1/A' \oint V'_{gs}\, ds'$, where the primes indicate that the quantities are computed from the map.

Since

$A' = m^2 A$,

$ds' = m\,ds$ and

$V'_{gs} = \frac{V_{gs}}{m}$,

we get

$$q = \frac{1}{A} \oint V_{gs}\, ds$$

$$= \frac{m^2}{A'} \oint m V'_{gs} \frac{ds'}{m} = \frac{m^2}{A'} \oint V'_{gs}\, ds'$$

$$= m^2 q'$$

The geostrophic vorticity can be computed from the conformal map and multiplied by m^2 to give the actual vorticity.

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APPENDIX - 6.

The Scale Variation of the three Conformal Projections

The values of m, the scale variation for the different conformal projections are given in the following table for 5° latitude intervals. These values are also shown graphically in Fig.16 which can be used to ascertain,

(i) the scale distortion at any given latitude, and

(ii) the latitudes within which the distortion is confined to given limits, say, 1.10 and 1.15 and so on.

The most appropriate projection is one that involves the least amount of distortion for a given region.

vi)

Table showing the values of scale for the recommended Conformal Projections

Latitude	Mercator's with standard parallels at $22\frac{1}{2}$°N and $22\frac{1}{2}$°S	Lambert's Conformal conical with standard parallels at 30°N 60°N	Lambert's Conformal conical with standard parallels at 10°N 40°N	Polar Stereographic with standard parallel at 60°N
90 N	∞	∞	∞	0.93
85 N	10.60	1.57	3.19	0.93
80 N	5.32	1.29	2.16	0.94
75 N	3.57	1.16	1.72	0.94
70 N	2.70	1.08	1.48	0.96
65 N	2.19	1.03	1.32	0.97
60 N	1.85	1.00	1.21	1.00
55 N	1.61	0.98	1.13	1.02
50 N	1.44	0.97	1.07	1.05
45 N	1.31	0.97	1.03	1.09
40 N	1.21	0.97	1.00	1.13
35 N	1.13	0.98	0.98	1.18
30 N	1.07	1.00	0.97	1.24
25 N	1.02	1.03	0.97	1.31
20 N	0.98	1.06	0.97	1.38
15 N	0.96	1.10	0.98	1.48
10 N	0.94	1.15	1.00	1.58
5 N	0.93	1.21	1.03	1.71
0	0.92	1.28	1.06	1.86
5 S	0.93	1.35	1.11	2.04
10 S	0.94	1.43	1.16	2.26
15 S	0.96	1.55	1.23	2.52
20 S	0.98	1.76	1.32	2.84
25 S	1.02	1.96	1.42	3.23
30 S	1.07	2.20	1.55	3.73
35 S	1.13	..	1.71	4.38
40 S	1.21	..	1.92	5.22

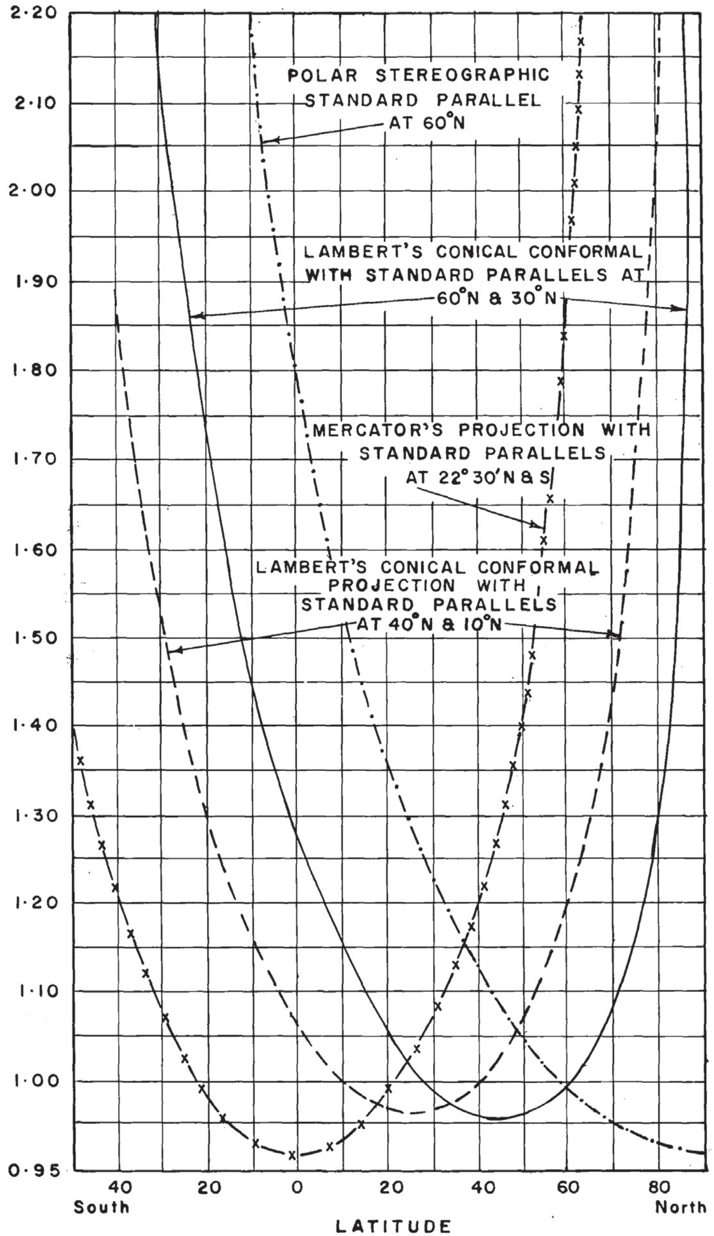

FIG. 16: THE GRAPHS FOR SCALES OF THE THREE CONFORMAL PROJECTIONS