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### INDIA METEOROLOGICAL DEPARTMENT

## FORECASTING MANUAL

PART II

METHODS OF ANALYSIS

4 ANALYSIS OF WIND FIELD

BY

R. N. KESHAVAMURTY

ISSUED BY

THE DEPUTY DIRECTOR GENERAL OF OBSERVATORIES (FORECASTING) POONA - 5

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#### FORECASTING MANUAL

Part II. Methods of Analysis

4. Analysis of Wind Field

by

R.N. Keshavamurty

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#### 1 Introduction

1.1 Wind, like other meteorological parameters, is observed at discrete points but its field is a continuum. In order to get an idea of this field we interpolate between the observations. If we are considering a scalar field, we draw lines of equal value of the parameter or isolines (eg. isobar, isotherm) at convenient intervals (of the parameter). These isolines form certain patterns; by physical considerations and experience we have come to associate weather with certain of these patterns. The process of drawing these isolines is called analysis and its main aim is the delineation of these patterns and inferring the associated weather. The main basis of synoptic forecasting is that these patterns and the associated weather often move from one region to another.

1.2 For numerical computations and also numerical weather prediction, the values of the wind at grid-points (say at every 5 degree or 2 degree points) are required. From a chart on which the isolines have been drawn, it is easy to do this further interpolation at grid points. However, this interpolation at grid points based on data at observing stations is generally done on the computer and is called 'objective analysis' Discussion of this type of analysis is beyond the scope of this report.

1.3 Wind is a vector and is described by two scalar quantities like the direction from which it blows and its speed. A complete analysis of the wind field requires analysis of the two scalar fields describing it. Lines of equal direction, called isogons, and lines of equal speed, called isotachs, can be drawn; and these describe the wind field completely. Another set of two scalar fields that can be used to describe the wind field is the westerly (u) and the southerly (v) components of wind. However, it is customary to analyse the wind field with the help of streamlines and isotachs. A streamline is a line parallel everywhere to the instantaneous direction of the wind. It refers to an instant of time, like a snapshot. The equation to a streamline is

$$\frac{dy}{dx} = \frac{v}{u} \quad (Fig.1) \quad \dots (1)$$

The streamline is different from the trajectory which is the path traced by a parcel as it moves. The two coincide only when the flow is strady.

#### 2. Evolution of Wind Analysis

2.1 The wind field is represented and analysed in different ways. The winds can be plotted at the stations as vector arrows with their directions showing the wind direction and their lengths proportional to the speeds. A better representation of wind speed is the internationally accepted method of representation by barbs and solid pennants, a full barb representing ten knots, a half barb five knots and a solid pennant 50 kt. It is worthwhile plotting the arrows (direction) with the aid of a protractor, if the analysis is used for computations. A visual inspection of such a plotted chart gives an idea of the wind field. But there is a limit to the information that can be assimilated by the eye, particularly when the density of network (of observations) is small.

2.2 A quick analysis is done by drawing the main flow-lines or rudimentary streamlines - just showing the major systems. These streamlines may also be discontinuous as in Fig. 2. It is a single set of lines drawn tangential to the winds. They serve the purpose of focussing attention on the main synoptic systems in the chart. In this type of analysis an idea of the speed can be obtained by the qualitative packing of the streamlines, i.e. by drawing more streamlines (greater packing) in regions of stronger winds and less number of streamlines (sparse packing) in regions of weaker winds. A justification for this procedure follows. The wind vector can be written as

$$W = \mathbf{i}\mathbf{k} \times \nabla \Psi + \nabla \mathcal{X} \qquad \dots (2)$$

after Helmholtz, where  $\Psi$  is the streamfunction and  $\chi$ , is the velocity potential. The first term gives the non-divergent part of the wind and the second the divergent or irrotational part of the wind. It is generally known that the non-divergent part of the wind is an order of magnitude larger than the divergent part. So, for the present let us assume

$$\mathbb{V} \approx \mathbb{I} \mathbb{k} \times \nabla \mathcal{V}$$
 ...(3)

If we draw lines of constant  $\psi$  or streamlines, the wind direction will be parallel to these lines and the speed will be inversely proportional to the distance between the streamlines (Fig.3). The wind speed is  $V = \frac{\partial \psi}{\partial n}$  Between two streamlines  $\partial \psi$  is constant, so that the speed is inversely proportional to the distance  $\partial \eta$  between them.

2.3 The divergent part of the wind, though small, is very important in meteorology, as all vertical motion and weather depend upon it. It also produces or destroys the non-divergent part. Also, the packing of the streamlines according to wind speed is qualitative. Therefore, it would be advantageous to analyse the speed-field separately by drawing isotachs. The streamline-isotach analysis is a complete analysis of the wind field (Fig. 4). It is sometimes referred to as kinematic analysis. We shall deal with this method in detail in later sections.

#### 3. Differential Properties of the Wind Field

3.1 At an instant the  $\mathcal{U}$  and  $\mathcal{V}$  components of the wind are functions of  $\mathbf{x}$  and  $\mathcal{Y}$  (at a particular level, z = constant).

u = u(x,y); v = v(x,y)

These can be written as Taylor expansions about an arbitrary origin (subscript '0').

 $\mathcal{U} = \mathcal{U}_0 + \left(\frac{\partial u}{\partial x}\right)_0^{\chi} + \left(\frac{\partial u}{\partial y}\right)_0^{\chi} \mathcal{Y} + \text{higher order terms } \dots (4)$ 

$$\mathcal{V} = \mathcal{V}_{0} + \left(\frac{\partial \mathcal{V}}{\partial \mathbf{x}}\right)_{0}^{\mathbf{x}} + \left(\frac{\partial \mathcal{V}}{\partial \mathbf{y}}\right)_{0}^{\mathbf{y}} \mathcal{Y} + \text{higher order terms } \dots (5)$$

We shall consider only first order terms; this will be valid close to the origin. Now introducing the variables

 $D = \text{divergence} = \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)_{0}$  $\mathcal{E} = \underset{(\text{vertical component})}{\text{vertical component}} = \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)_{0}$ 

$$F = deformation (stretching) = \left(\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}\right)_{0}$$
  

$$F' = deformation (shearing) = \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)_{0}$$

into expressions (4) and (5) we get

$$\mathbf{u} = \mathbf{u}_{0} + \frac{1}{2} \mathbf{D} \mathbf{x} - \frac{1}{2} \mathbf{y} + \frac{1}{2} \mathbf{F} \mathbf{x} + \frac{1}{2} \mathbf{F} \mathbf{y} \qquad \dots (6)$$

$$\mathbf{v} = \mathcal{V}_{0} + \frac{1}{2} D\mathbf{y} + \frac{1}{2} \mathbf{\xi}^{\mathbf{x}} - \frac{1}{2} \mathbf{F}\mathbf{y} + \frac{1}{2} \mathbf{F}^{\mathbf{x}} \dots (7)$$

These relations show that the wind motion is a combination of translation, divergence, vorticity (or rotation) and deformation. These properties and associated streamlines and isotachs are described in Table 1. In nature, various combinations of these types of flow occur (Figs. 5 and 6).

#### 4. Some Common Patterns in the Wind Field

4.1 The simplest type of flow we come across is a straight current. e.g. the trades and westerly current (when not perturbed). Since we know that divergence is small, the isotachs tend to be more or less parallel to the streamlines or cut them only at small angles.

4.2. The most important common patterns are those around singular points. A singular point is defined as a point where the wind speed is zero. Centres of pure rotation, divergence and deformation discussed in the earlier section or different combinations of these, are examples of singular points. We shall consider the common combinations that occur in nature.

i) Vortices: The main flow component of these patterns is rotation. Generally divergence (convergence) and/or deformation is superposed on pure rotation. In the case of rotation plus superposed divergence/convergence, the stream-lines are logarithmic spirals (Figs6 a and b). The angle of outflow/ inflow, d, is given by

$$\tan \alpha = \frac{\text{divergence}}{\text{vorticity}} \qquad \dots (8)$$

and is generally small. The inflow (or outflow) is generally marked only in the frictional layer. Above the frictional layer, opinion is divided whether pure rotation should be shown or inflow (or outflow) should be shown. If deformation is also superposed on this pattern of rotation plus superposed divergence (and convergence), we get patterns as in Figs. 6 c. and 6 d. All these are referred to as vortices.

4.3 The singular points in the wind field-vortices and neutral points - are singular points in the isogon field also. They are meeting points of isogons. Fig. 8 a,b,c show the isogon patterns associated with clockwise (anticyclone in the northern hemisphere) and anticy ockwise (cyclone in the northern hemisphere) vortices and neutral points respectively. The isogon values increase in a clockwise sense in the case of vortices and in an anticy ockwise sense in the case of neutral points.

4.4 The westerly current and the trades are seldom straight currents. They are perturbed and show wave patterns. The troughs and ridges usually have a minimum of speed along the axis and maximum ahead and in the rear. Fig. 9 shows the streamline-isotach pattern associated with a typical westerly trough.

4.5 Asymptotes of convergence or divergence are lines to which neighbouring streamlines converge or from which neighbouring streamlines diverge respectively

(Fig. 10a and b). These are not necessarily areas of velocity divergence as that would depend upon the speed field also.

5. Practical Aspects of Streamline - Isotach Analysis

5.1 Since this type of analysis is generally done in the tropical region mercator projection maps are convenient.

5.2 It is necessary to draw the streamlines first and later do the isotach analysis as the latter analysis requires the streamline field. The streamlines are drawn by two methods:

i) the direct or free-hand method and

ii) the isogon method

The isogon method is superior to the free-hand method, but it is time-consuming. In operational offices it is convenient to draw the streamlines by the free-hand method. The streamlines are drawn everywhere parallel to the wind direction. They are continuous lines and can meet in regions of confluence. Though there is no restriction on the number of streamlines i.e. their packing, it is better to restrict the number of lines to a minimum in regions of weak winds in order to avoid undue emphasis of weak systems. In the surface friction layer, cyclonic indraft and anticyclonic outdraft are shown. Above the friction layer the position is not so clear (as mentioned earlier). Except in regions of very good data coverage where the inflow or outflow is conclusive, cyclones and anticyclones can be drawn as closed vortices. Anticyclones are marked 'A' and cyclones 'C' at the centre.

5.3 In the isogon method of streamline analysis, isogon analysis is made first. The numerical values of the wind direction are plotted at the stations (say in tens of degrees). Lines of equal direction or isogons (Fig. 11 a) are drawn by scalar interpolation, say at intervals of 30 degrees. Across each iscgon a number of short segments are drawn parallel to the wind direction corresponding to the isogon. These segments serve as additional observations of wind

direction. Now the line segments are connected by tangential curves to obtain streamlines (Fig. 11b).

5.4 After completing the streamline analysis, the isotachs are drawn. Though this is a scalar analysis, it is better to keep in mind the patterns of isotachs associated with typical streamline patterns mentioned in the earlier sections. Isotachs are drawn as smooth lines. As in any other scalar analysis minor kinks in the lines are generally avoided. Elongated speed maxima occur in the major currents like the trades and the westerlies (Fig. 12). There can be more than one maximum in a broad current. There are minima of speed in the region of singular points (they are defined as centres of zero speed). The isotachs can be drawn at intervals of 20 kt in main current, and at 10 kt near singular points. Near singular points the isotachs are circles; but when some deformation is also superposed, the isotachs are in general ellipses. Near neutral points isotach pattern shown in Fig. 7 d has been suggested by Palmer et.al.

5.5 It is necessary to maintain continuity in time and in the vertical. For maintaining time continuity the analysed chart of the previous hour (corresponding to the same level) is put below the current chart to be analysed and they are kept on a light-table\*. The earlier positions of centres of anticyclones, cyclones, neutral points and axes of troughs and ridges are marked (lightly) on the current chart. The positions of the centres and axes on the current chart should be consistent with the earlier ones. In fixing these centres or axes, use is also made of the contour heights (except in equatorial regions) and also of the fact that there is a speed minimum at singular points.

5.6 Continuity in the vertical has also to be maintained, as most synoptic systems of significance extend through some depth of the atmosphere. For this purpose, the chart of the higher level (say 700 mb) is put above the lower level

\* Analysis can be made on an acetate overlay also.

chart (say 850 mb) on a light table It is checked whether the slope (or tilt) of lows and highs is consistent with the fact that lows slope towards lower temperatures and highs towards higher temperatures\*. Cold lows increase in intensity with height and warm lows decrease in intensity with height and may be replaced by anticyclones aloft (eg. the heat low over northwest India - Pakistan). An exception is the tropical storm (which is warm-cored) where (due to the vertical momentum exchange in the cumulus-cumulonimbus clouds) the intensity (cyclonic vorticity) remains practically same upto great heights. Warm (cold) anticyclones increase (decrease) in strength with height. A cyclone (antityclone) which is strongest at any level may appear as a trough (ridge) above or below.

5.7 The wind observations (like any other meteorological parameter) contain errors (due to human failure and faulty communication). They can be checked by climatological ranges and by time and space continuity i.e. by comparing with neighbouring and earlier data. Large changes in the wind field in time and space can occur only in association with well-marked systems. Another aid for checking the data is the relation between wind and pressure (or contour height) and also the correlation with weather. If any radical change is to be made in the earlier patterns on the basis of meagre data, there is a case for checking the data. Of course, in regions of light winds, the directions can change rapidly with time and are not of much significance (eg. in the transition level around 400 mb during the southwest monsoon).

5.8 It is advantageous to have an ideard the mean wind field of the region of analysis. An idea of the synoptic climatology of the major systems in the region is also necessary. Only a few examples follow:- The monsoon trough and monsoon depressions generally extend from the surface up to 500 mb (occasionally when strong up to 400 mb) and normally slope wouthwards with height as the tempe-

<sup>\*</sup> This assumes a relationship between wind shear and temperature field based on the geostrophic relation. Its validity in the equatorial region is questionable.

rature gradient in the monsoon is from the south to the north (colder temperature is to the south). Tropical storms are vertical and the cyclonic circulation (on synoptic scale) extends almost to 300 mb with marked inflow only in the lowest levels, outflow around 200 mb and little inflow or outflow in between. Further elaboration of this aspect is beyond the scope of the present report.

#### 6. Auxiliary Aids to Analysis

6.1. Vertical time-sections: The passage of troughs or other systems across a station can be identified on a time-section by the wind shift and contour height changes. In data-sparse regions this information can be used for analysis.

6.2 Satellite cloud data: Tropical storms can be easily identified in satellite cloud pictures. The centres given by them can be used on wind charts. For lower intensity systems like depressions and lows, the correlation between the cloud and wind fields is not so unique and is still a subject of research. However, low level cumulus spirals give an indication of circulation centres. At the cirrus level, cirrus shearing from cumulonimbus tops gives a qualitative idea of the wind direction and speed. The jet stream has some characteristic long patches of cloud associated with it. These cloud information can be of help in wind analysis in data-sparse regions. Satellite-derived wind information for the upper tropospheric levels is now available on an operational basis, as a routine.

6.3 Facsimile charts from neighbouring regions are useful for analysis near the borders of the region of analysis.

#### 7. Main Uses of Wind Analysis

7.1 The main use of analysis (as mentioned earlier) is the association of weather with certain upper air patterns. Generally upward motion and weather are associated with regions of convergence in the field of cyclonic circulations, troughs and asymptotes of convergence in the lower troposphere and with regions ahead of troughs and other regions of divergence in the upper troposphere. As the

inaccuracies of wind data introduce large errors in the computation of divergence, it is difficult to use the wind analysis directly for the computation of divergence and vertical motion. A better way would be to compute the vorticity field ( $\boldsymbol{\zeta}$ ) and judge the velocity divergence ( $\nabla_{\boldsymbol{H}} \bullet \boldsymbol{W}$ ) from the vorticity equation

$$\frac{1}{\xi_0} \frac{d\xi_0}{dt} = -\nabla_{H} \bullet W \qquad ...(9)$$

where  $\xi_{\mathbf{a}} = \xi + f$  where f is the coriolis parameter. The gain of cyclonic vorticity of a parcel generally occurs in regions of strong gradients of vorticity i.e. near lows, troughs and jet maxima.

7.2 Vorticity is one of the important quantities that can be derived from the wind field. Wind direction and speed can be picked out at (5 degree or 2 degree) grid points from the wind analysis and the vorticity can be computed as follows:

$$\zeta_{0} = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$$

$$\zeta_{0} = \frac{(v_{3} - v_{4}) - (u_{3} - u_{1})}{2 d} \quad (Fig. 13) \quad \dots (10)$$

Another very useful derived quantity is the streamfunction (  $\psi$  ). It can be obtained from the vorticity field by solving the Poisson equation

$$\zeta = \nabla^2 \psi \qquad \dots (11)$$

This is usually solved on the computer by the method of relaxation.

7.3 Another use of the analysed wind field is to obtain the height field from it by using the balance equation and supplement or check the actual height field. However, for these purposes the wind field itself is generally analysed 'objectively' on the computer.

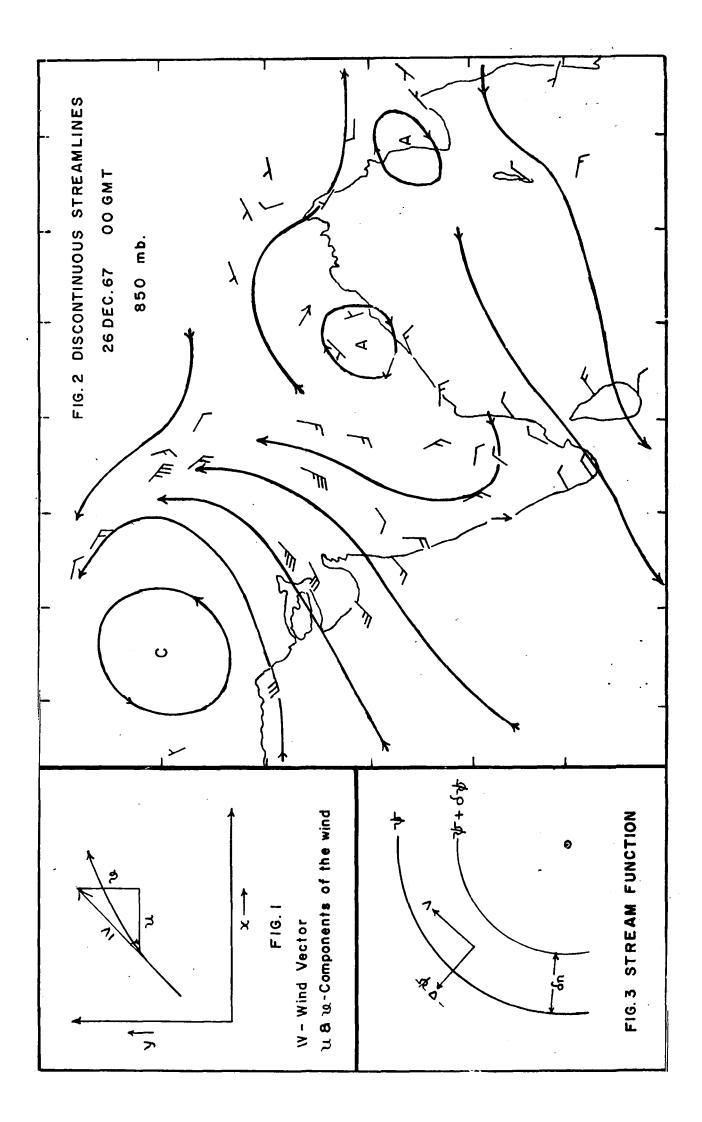
7.4 In aviation forecasting the latest wind field along the route is required. In low latitudes, winds from the direct analysis will be quite useful for this purpose.

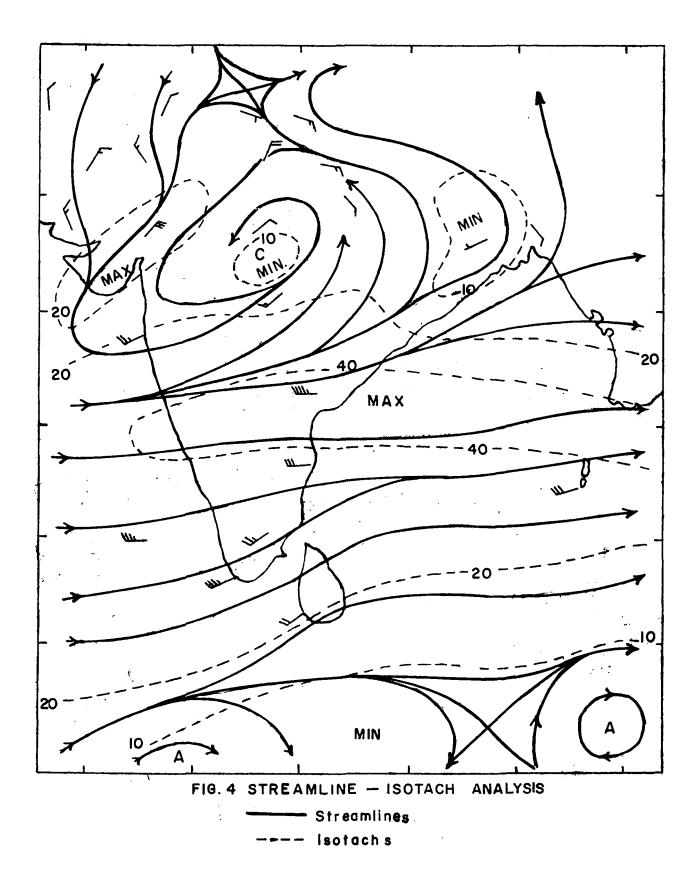
11 <u>TABLE - 1</u> 	Deformation (shearing)	$u = l_2 F' y$	$v = \frac{1}{2} F'x$	$\frac{dy}{dx} = \frac{y}{u} = \frac{x}{y}$	2 2 X - y = const.	A fam lar h their tilte	$V_{u+v}^{z} = const.$ $v_{u+v} = const.$ $r_{x+y} = const.$ A family of concen- tric circles with centre at the	origin. Fig. 5(g)
	Deformation (stretching)	$u = \frac{1}{2} Fx$	$v = -k_2 Fy$	$\frac{dy}{dx} = \frac{y}{u} = \frac{-y}{x}$	i.e. $\mathbf{X}\mathbf{y} = \text{const.}$	A family hyperbola axes are as axes o and contr	$\sqrt{u^2 + v^2} = const.$ or $\frac{2}{x} + \frac{2}{y} = const.$ A family of concentric circles with the centric tre at the origin. The	two types of deforma- tion can be combined by a suitable choice of the coordinates. Fig. 5(f)
	Pure Rotation	$u = -\frac{1}{2} \xi_y$	$v = \frac{1}{2} \xi x$	$\frac{dy}{dx} = \frac{2t}{u} = -\frac{x}{3}$	i.e. $X + y = const$	A family of concentric circles with the cen- tre at the origin. The rotation is either clockwise or anti- clockwise.	$\sqrt{u^2 + v^2} = \text{const.}$ or $2 + y^2 = \text{const.}$ A family of concentric circles with the tric circles with the centre at the origin.	Fig. 5(d) and (e)
	Pure Divergence	$u = \frac{1}{2} Dx$	$v = \frac{1}{2} 0y$	<u>वित्र</u> = <u>ग</u> = <u>ग</u> = <u>म</u>		A family of straight lines passing through the origin - converging or diverging according to the sign of the slope.	$\sqrt{2} + \frac{2}{w} = const.$ or $2 + 3 = const.$ A family of concentric circles with the centric tre at the origin.	Fig. 5(b) and (c)
	Pure Translation	$n = n^{\circ}$	$v = v_{o}$	$\frac{dy}{dx} = \frac{y}{u} = \frac{y_{0}}{u_{0}}$	= const.	A family of parallel straight lines	The speed is everywhere the same	Fig. 5(a)
               		า	v				Equation to the isotachs and shape of the isotachs	

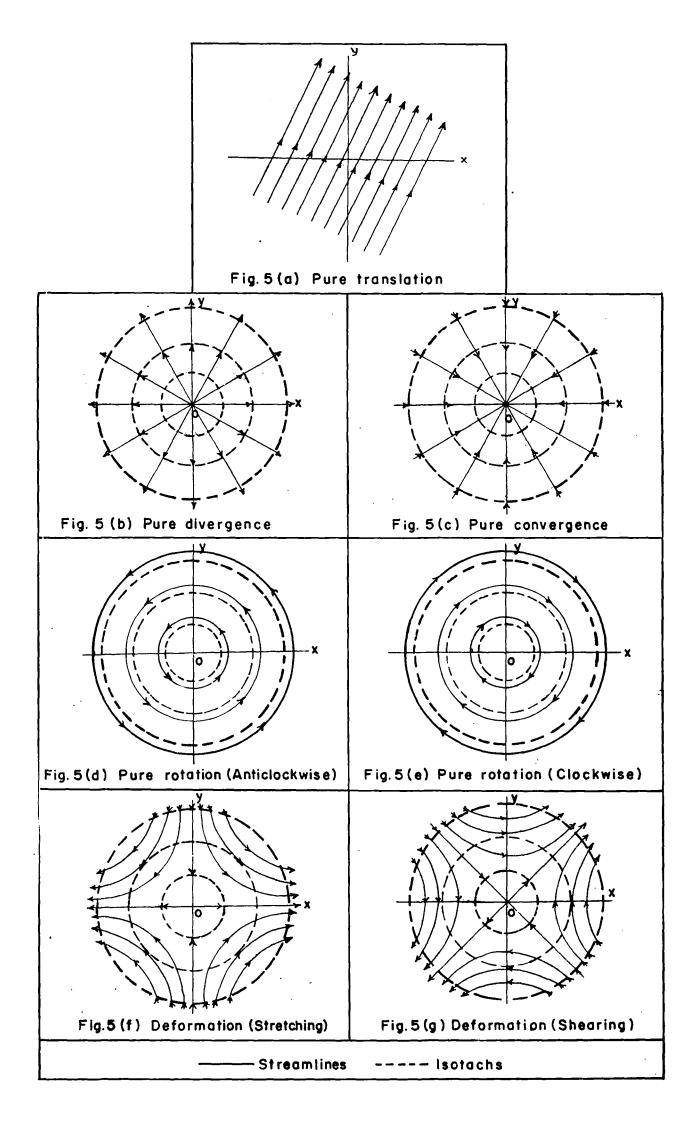
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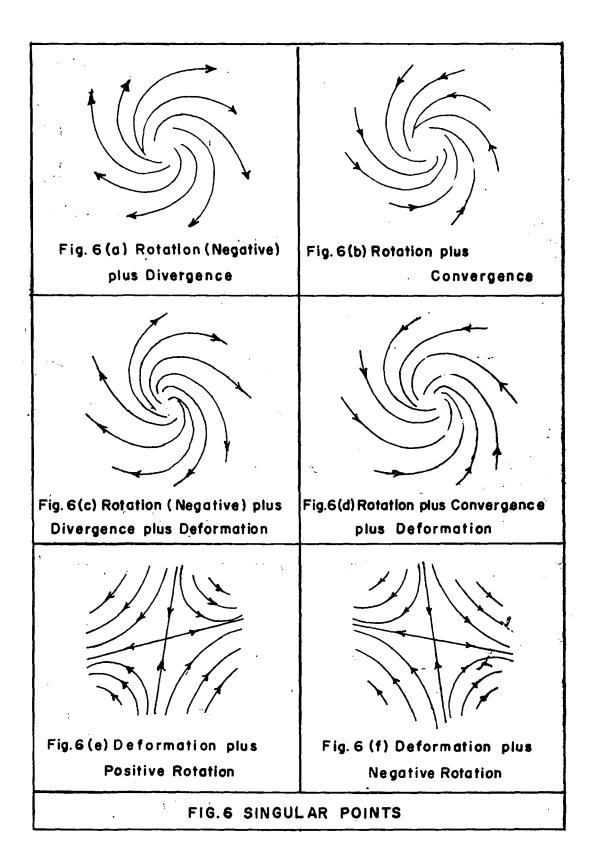
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# DIAGRAMS









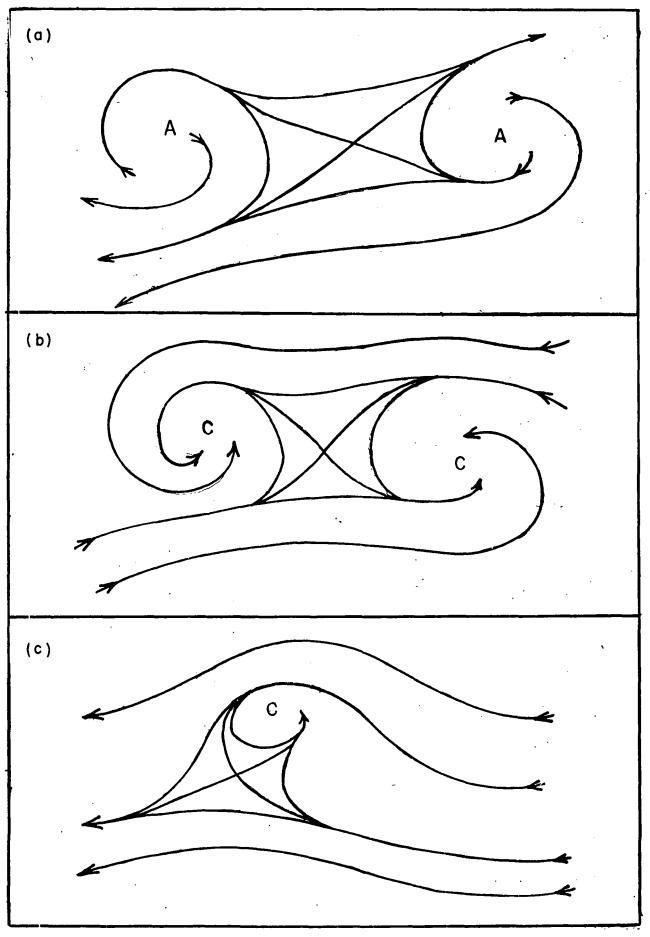


FIG.7 NEUTRAL POINTS

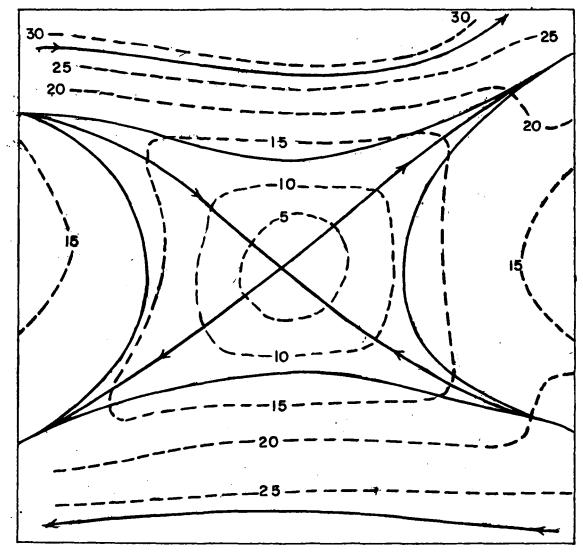
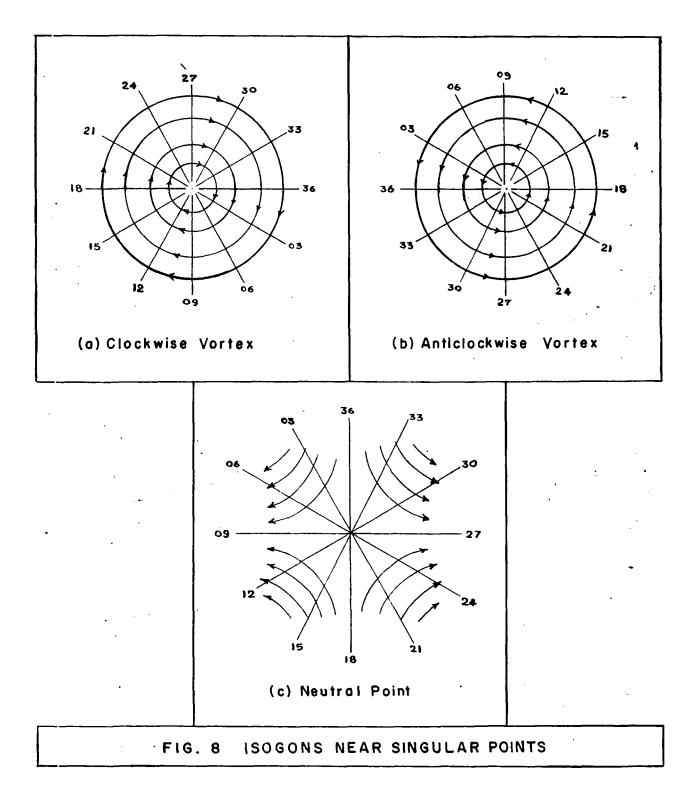


FIG.7(d) NEUTRAL POINT (Copied from Fig. 4-14 of Ref. 1)



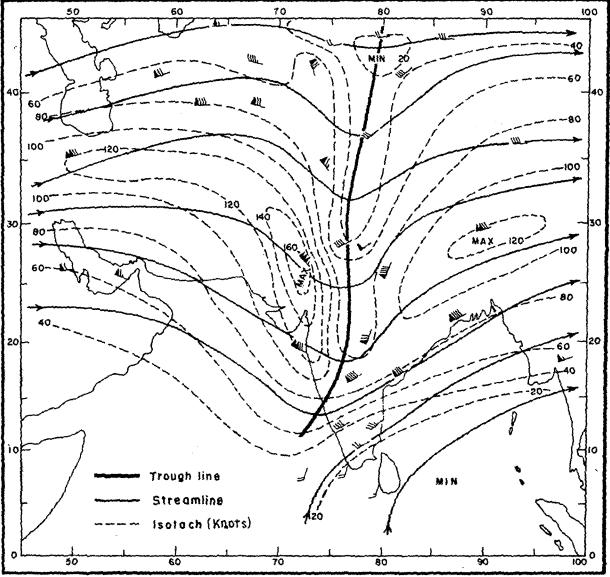
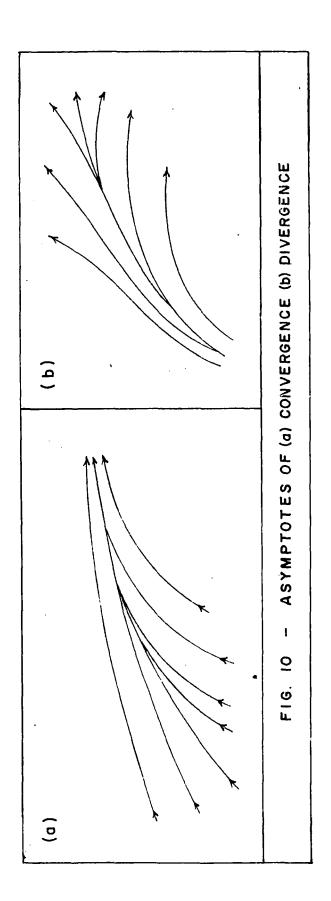
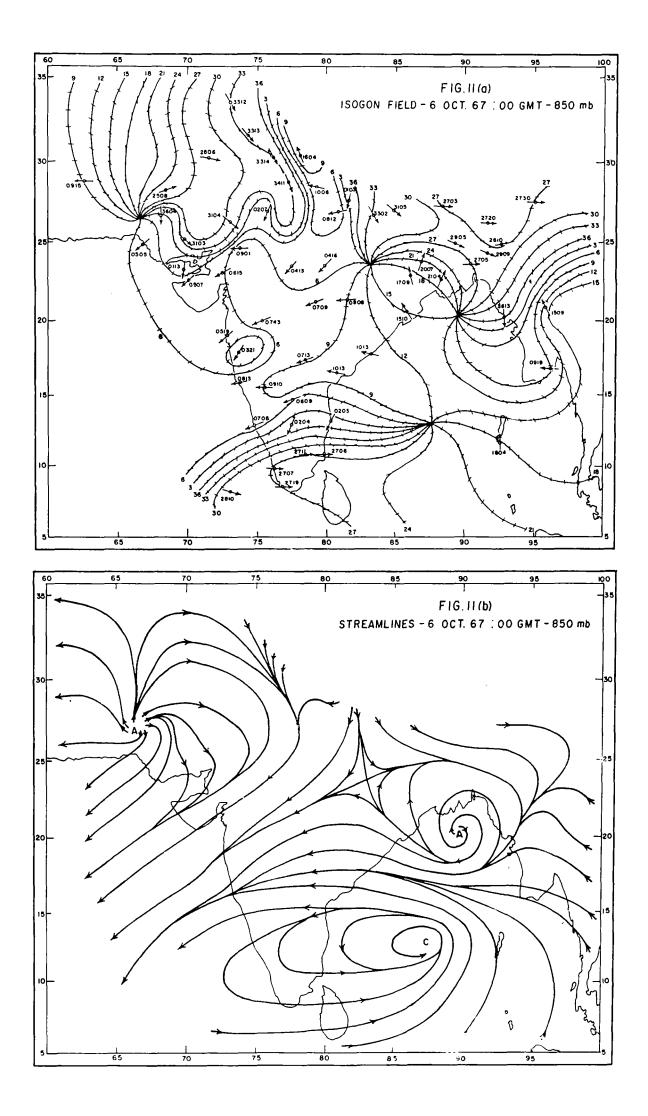
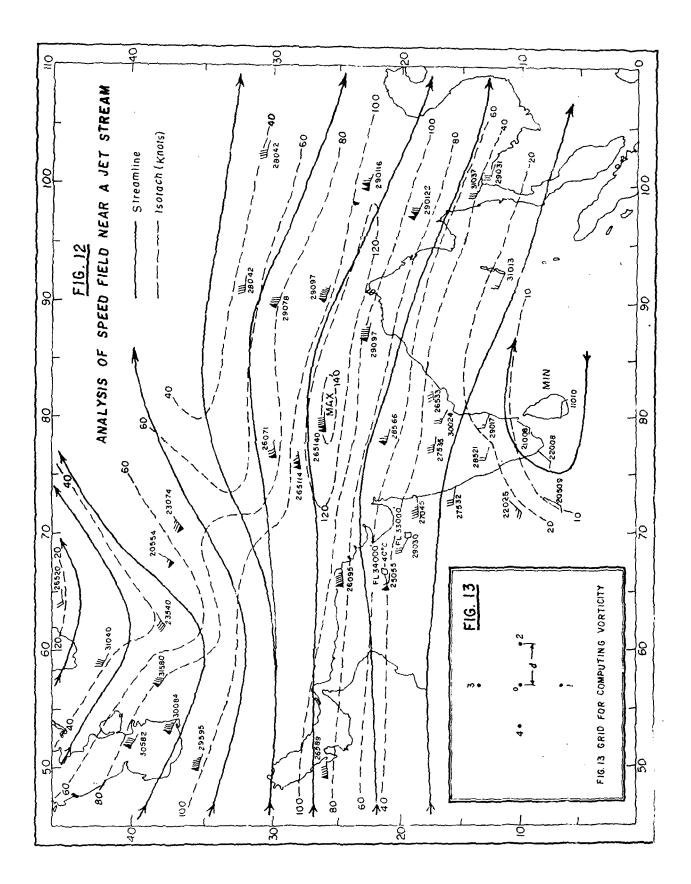


FIG. 9 TYPICAL TROUGH LINE







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