Kinematics of wind and pressure fields

⇔ Kinematics comes from the Greek word meaning "motion".
By kinematics we mean a description of the motion of a particular field without regard to how it came about or how it will evolve.

⇔ Kinematics is a branch of dynamics that deals with aspects of motion apart from considerations of mass and force - motion of objects without reference to forces which cause motion

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2021-22

Kinematics of wind field

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In the Cartesian coordinate system, an air parcel's location in two dimensions is defined as (x, y). Depending upon our choice of vertical coordinate, an air parcel's vertical position may be defined as z or p.

Wind, therefore, is simply defined as the change in the air parcel's location with time $\frac{d\vec{\mathbf{r}}}{dt} = \vec{\mathbf{V}}$, where $\vec{\mathbf{r}} = x\hat{\mathbf{i}} + \mathrm{y}\hat{\mathbf{j}} + (z \text{ or } p)\hat{\mathbf{k}}$, $\vec{\mathbf{V}} = \frac{dx}{dt}\hat{\mathbf{i}} + \frac{dy}{dt}\hat{\mathbf{j}} + \frac{d(z \text{ or } p)}{dt}\hat{\mathbf{k}}$

Few types of fluid trajectories: can be demonstrated using streamlines or trajectories

$\Rightarrow$ A streamline represents a line that is tangent (or parallel) to the wind at a given location.

$\Rightarrow$ A trajectory represents the path that an air parcel follows through time

Kinematics of wind field: (a) divergence, (b) shearing deformation, (c) stretching deformation, (d) vorticity

Kinematics of the wind field

Kinematics of wind field: (a) divergence, (b) shearing deformation, (c) stretching deformation, (d) vorticity.

Representing these changes in terms of partial derivatives,

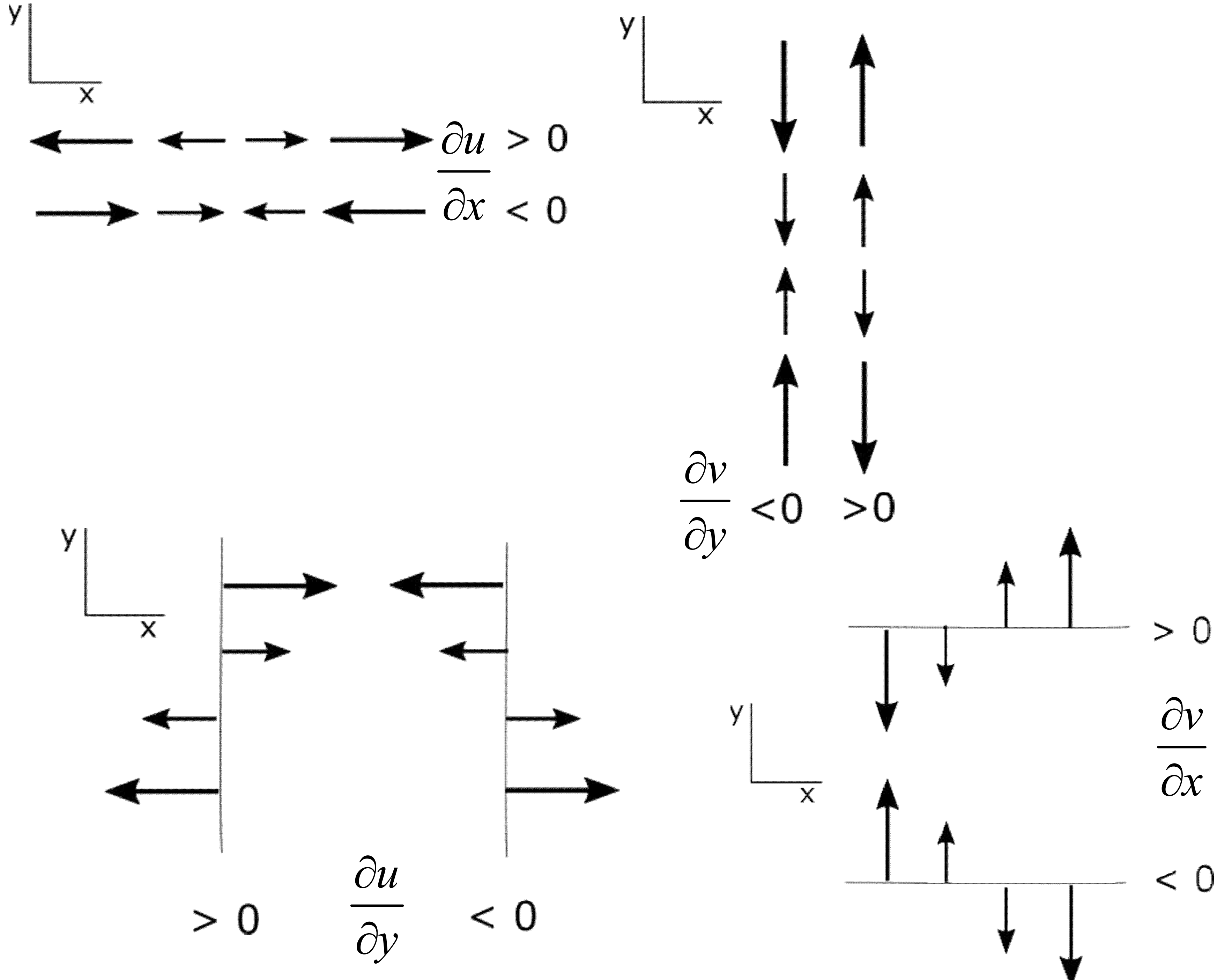
$$\frac{\partial u}{\partial x}, \frac{\partial v}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial y}$$

(1) Divergence of the wind field $\delta = \nabla \cdot \vec{\mathbf{V}}_{\mathbf{H}} = \dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y}$

(2) Stretching deformation $= \dfrac{\partial u}{\partial x} - \dfrac{\partial v}{\partial y}$

(2) Shearing deformation $= \dfrac{\partial v}{\partial x} + \dfrac{\partial u}{\partial y}$

(4) Vorticity $= \dfrac{\partial v}{\partial x} - \dfrac{\partial u}{\partial y}$



y
x
$\frac{\partial u}{\partial x}$ > 0
< 0
y
x
$\frac{\partial v}{\partial y}$ <0 >0
y
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> 0 $\frac{\partial u}{\partial y}$ < 0
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> 0
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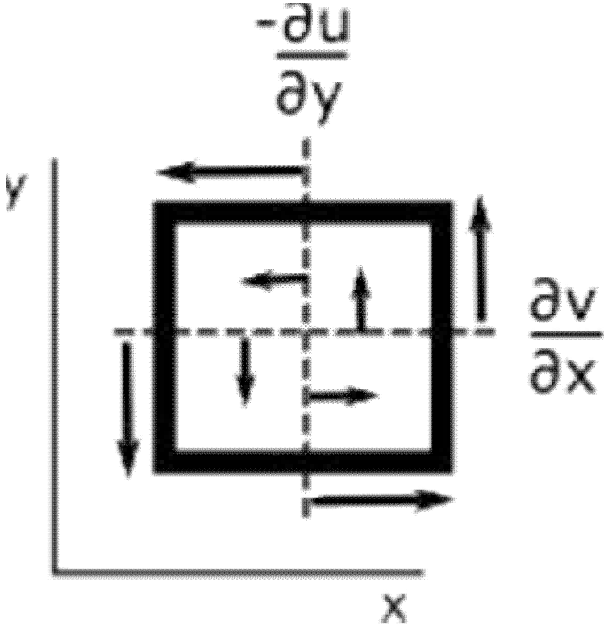
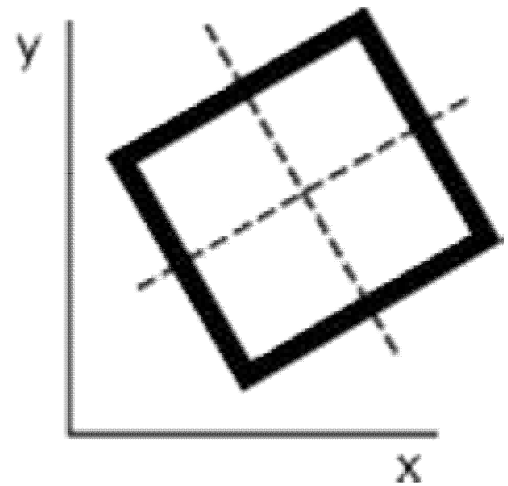
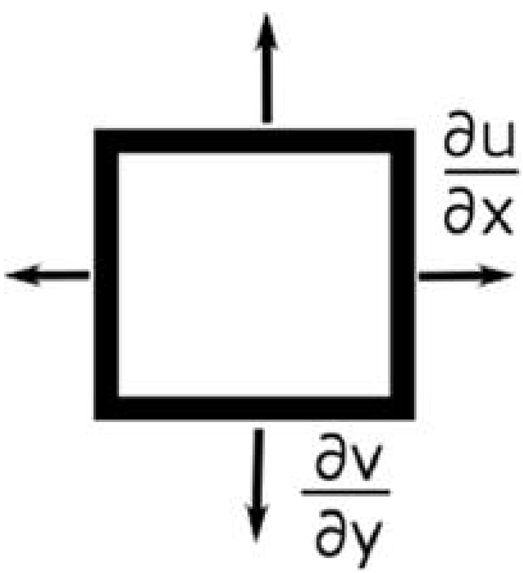
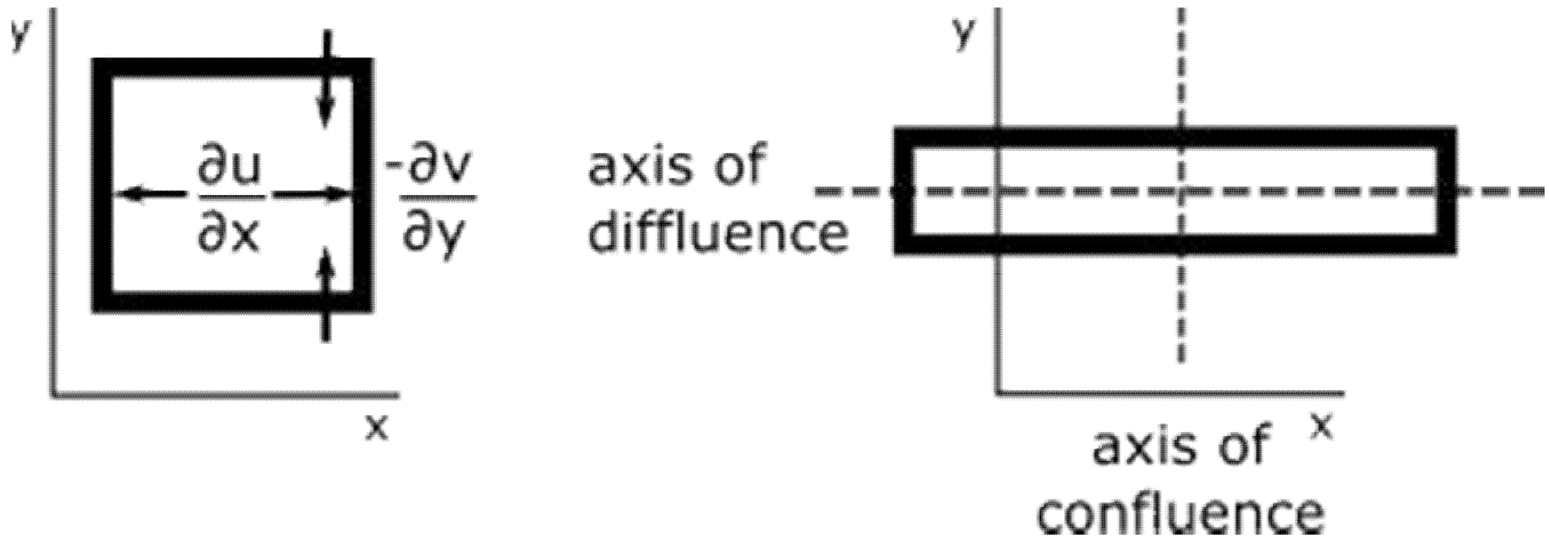




$$\text{Vorticity} = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$$

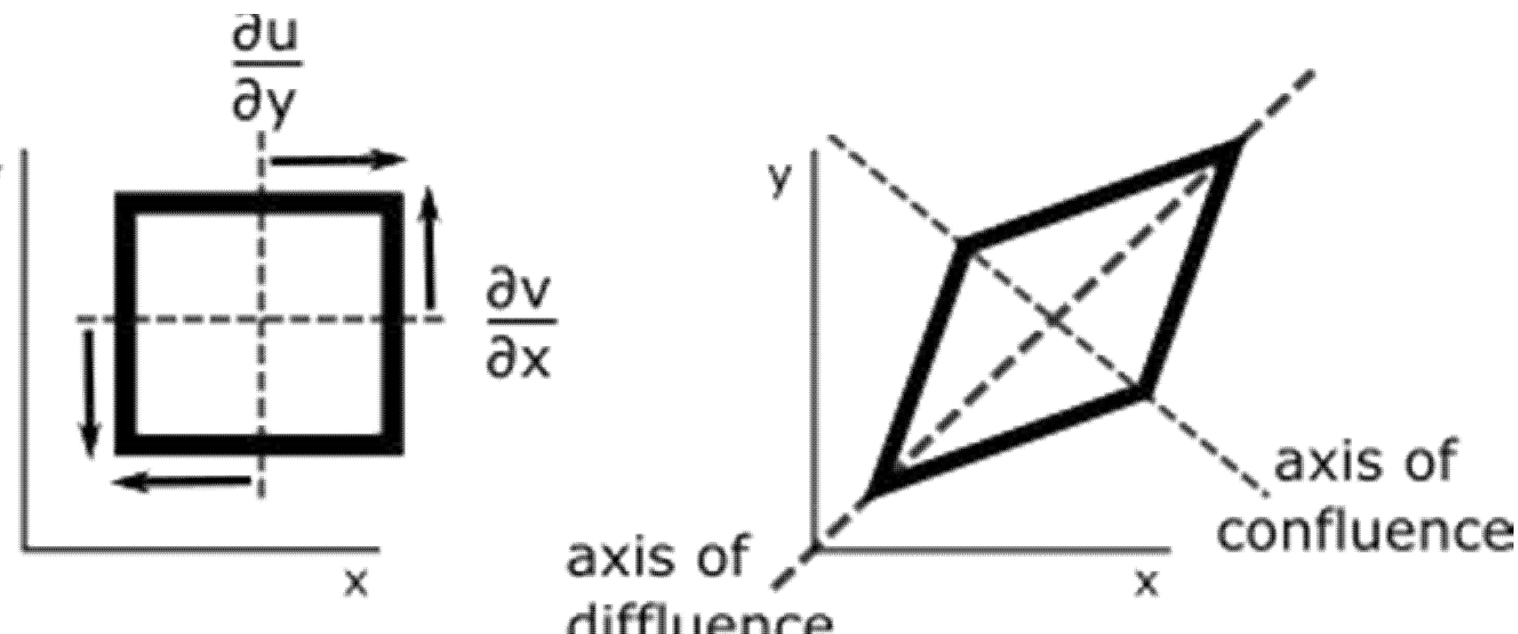


$$\text{Divergence} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$$



$$\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}$$

Stretching deformation



$$\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}$$

Shearing deformation

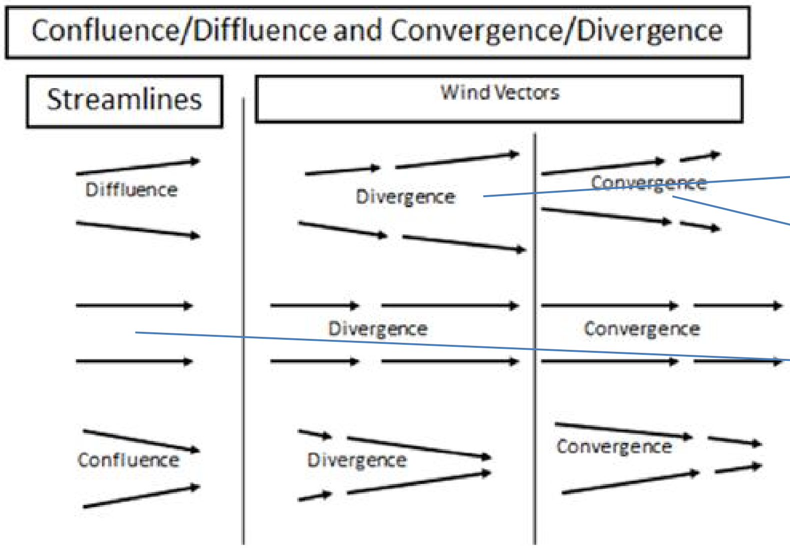
Streamlines

⇒ Streamlines are lines that are everywhere parallel to the velocity vectors at a fixed time

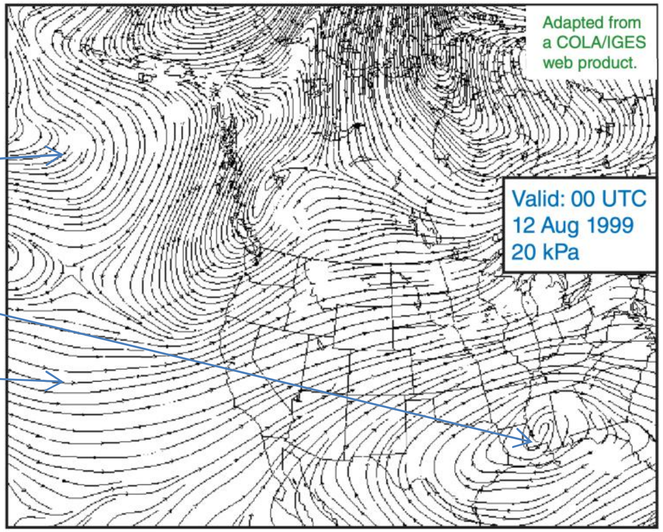
⇒ Streamlines consider the direction of the velocity but not the speed

⇒ Streamlines generally change from one time to the next

⇒ Confluence is when streamlines come together. Diffluence is when they move apart



In a steady flow, streamlines and streamlines all coincide



⇒ Streamlines that are more densely packed together imply a faster wind speed

⇒ Streamlines that are more loosely packed together imply a slower wind speed

⇒ Streamlines do not intersect, but they may diverge from or converge to a point on the chart. This occurs most commonly with areas of high and low pressure near the surface

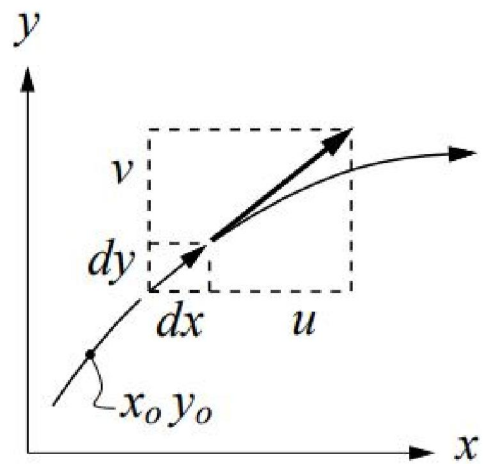
The differential equations of the streamline may be written as: $d\vec{\mathbf{s}} \times \vec{\mathbf{V}} = 0$

where $d\vec{\mathbf{s}}$ is an element of the streamline ($d\vec{\mathbf{s}} = dx\hat{\mathbf{i}} + dy\hat{\mathbf{j}} + dz\hat{\mathbf{k}}$)

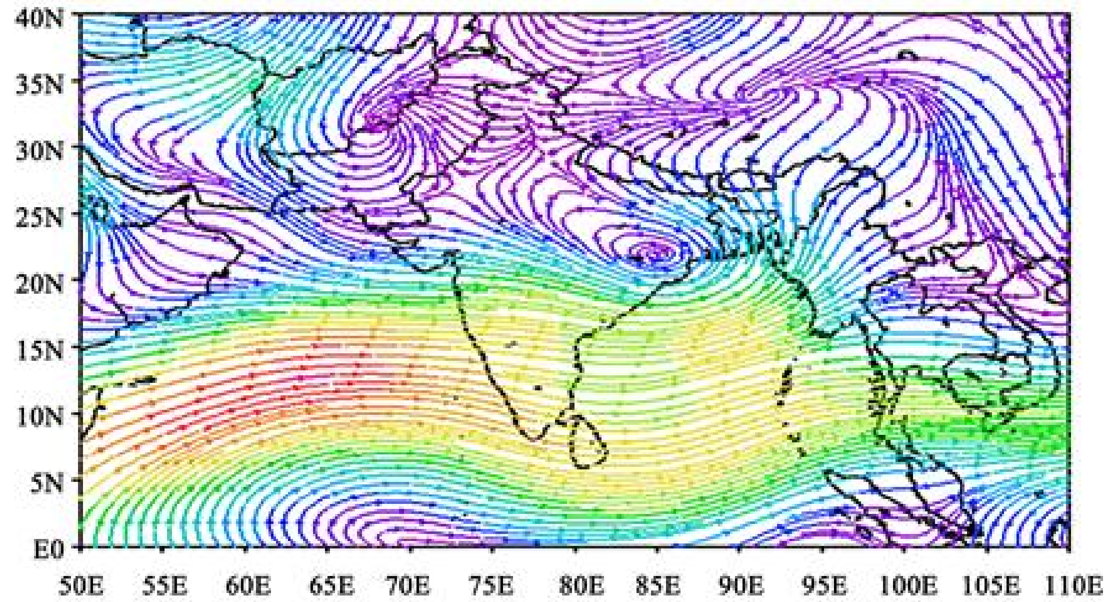
and $\vec{\mathbf{V}}$ the velocity vector; or in Cartesian coordinates,

$$d\vec{\mathbf{s}} \times \vec{\mathbf{V}} = (wdy - vdz)\hat{\mathbf{i}} + (udz - wdx)\hat{\mathbf{j}} + (vdx - udy)\hat{\mathbf{k}} = 0$$

Separately setting each component to zero gives three differential equations which define the streamline. In 2D we have $dz = 0$ and $w = 0$, and only the $\hat{\mathbf{k}}$ component of the equation above is non-trivial $\Rightarrow \dfrac{dy}{dx} = \dfrac{v}{u}$



2-D streamline



Why streamlines are good choice for tropics?

$$\frac{du}{dt} - \frac{uv\tan\varphi}{a} + \frac{uw}{a} = 2\Omega(v\sin\varphi - w\cos\varphi) - \frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\nabla^2 u$$

Let $f = 2\Omega\sin\varphi$ and $f' = 2\Omega\cos\varphi$

$$\frac{du}{dt} - fv + f'w + \frac{uw}{a} - \frac{uv\tan\varphi}{a} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\nabla^2 u$$

$L = 10^6$ m, $U, V = 10$ m s^{-1}, $H = 10^4$ m, $\varphi = 10^o$ (for simplicity), $f \approx 10^{-5}$ s^{-1}, W (small at large scales) = 10^{-2} m s^{-1}, $a =$ Radius of the earth $\approx 10^7$ m, $\rho =$ Air density = 1 kg m^{-3}. ν = dynamic viscosity of air $\approx 10^{-5}$ m^2 s^{-1} $\rightarrow$ $\nu\nabla^2 u \approx 10^{-12}$ m s^{-2} (tiny)

$$\frac{U^2}{L} - fU + f'W + \frac{UW}{a} - \frac{UV\tan\varphi}{a} = -\frac{1}{\rho}\frac{\delta p}{L} + F \quad \rightarrow \quad \frac{V^2}{R} + fV = -\frac{1}{\rho}\frac{\partial p}{\partial R}$$

$$10^{-4} \quad 10^{-4} \qquad 10^{-6} \qquad 10^{-8} \qquad 10^{-6} \qquad 10^{-4} \qquad 10^{-12}$$

⇒ This leaves us with gradient wind balance on synoptic length scales in tropics, since f is small.

⇒ Pressure gradient was chosen to balance the left side of the equation. It is about 100 Pa per 1000 km (i.e., 1 hPa for a length scale of 1000 km).

⇒ Since Coriolis acceleration is weak, synoptic scale pressure gradients are small, plotting surface pressure gradients in the tropics is not a particularly useful analysis in general.

⇒ Convergence and rotation of winds are important to tropical circulations. Therefore, streamline analysis is of greater use than plotting pressure on a constant height surface especially at low levels.

Trajectories - *a Lagrangian concept!*

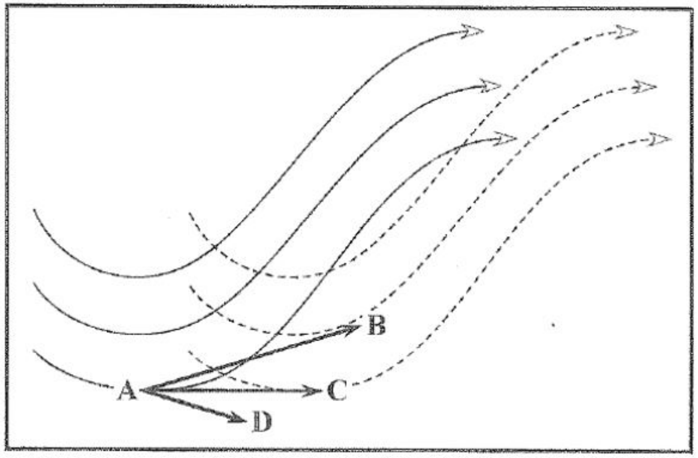
$\Rightarrow$ Trajectories are the actual paths of the moving air parcels, and indicate both the direction and velocity of air parcels over time.

$\Rightarrow$ Convergence is when the velocity of the air slows down in the direction of the streamline.

$\Rightarrow$ Divergence is when the velocity of the air speeds up in the direction of the streamline.

$\rightarrow$ streamlines are the path that an air parcel would follow if the wind did not change with time, while trajectories are the path that an air parcel follows while accounting for the fact that the wind does change with time

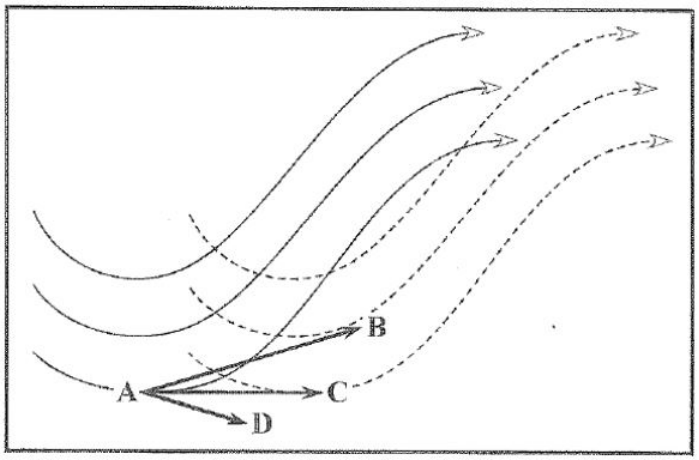
$\rightarrow$ Note also that while streamlines are often analyzed only using the horizontal wind, trajectories are typically analyzed using the fully three-dimensional wind



$$\vec{\mathbf{r}}(t_1) = \vec{\mathbf{r}}(t_0) + \int_{t_0}^{t_1} \vec{\mathbf{V}}(x, y, p, t)\ dt$$

A forward trajectory depicts where the air parcel moves to as time moves forward ($t_0 < t_1$) , while a backward trajectory depicts where an air parcel came from at previous times ($t_0 > t_1$)

Streamlines and forward trajectories are presented for an eastward-moving trough



⇒ Case 1 ($A \rightarrow D$): An air parcel moves slower than the trough. Starting at A, the air parcel initially moves due east. However, because the trough moves east faster than does the air parcel, the air parcel falls behind and ends up in the northwesterly flow behind the base of the trough.

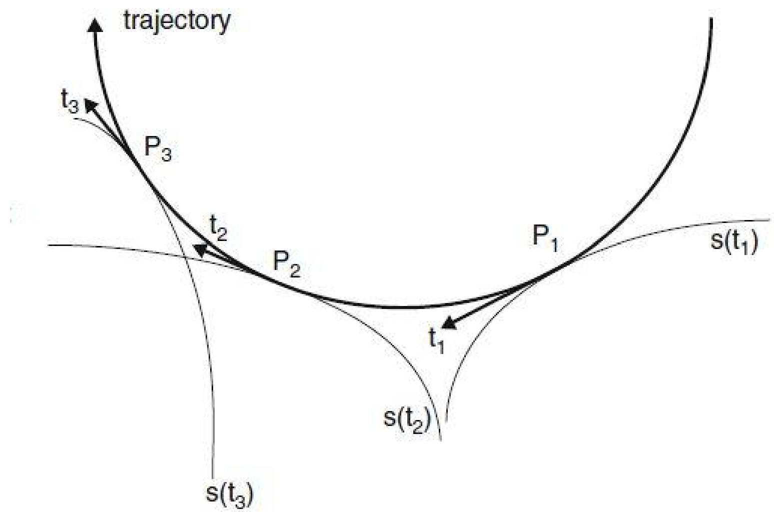
⇒ Case 2 ($A \rightarrow C$): Starting at A, the air parcel initially moves due east. Because the trough and air parcel move to the east at the same rate of speed, the air parcel is always located in the base of the trough where the wind is directed due east. As a result, its motion remains toward the east.

⇒ Case 3 ($A \rightarrow B$): An air parcel moves faster than the trough. Starting at A, the air parcel initially moves due east. However, because it does so faster than does the trough the air parcel ends up in the southwesterly flow ahead of the base of the trough.
→ As a result, its motion turns toward the east-northeast.

Blaton's equation

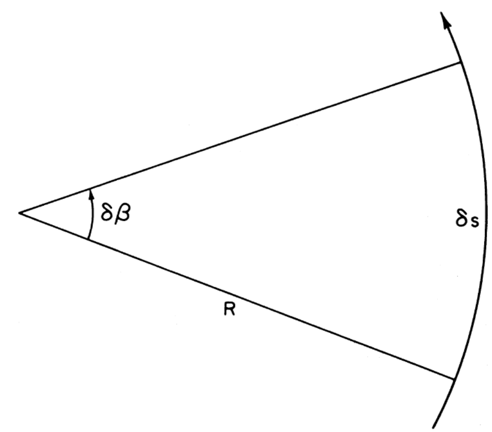
gives a relation between the radii of curvature of the streamline and the trajectory

$\Rightarrow$ At a given point $P(t)$, the unit tangent $\hat{\mathbf{t}}$ represents both the direction of this trajectory and the direction of the streamline touching the trajectory at this point.



$$\frac{\partial \hat{\mathbf{t}}}{\partial t} = V\left(\frac{1}{R_t}\hat{\mathbf{n}}_\mathbf{t} - \frac{1}{R_s}\hat{\mathbf{n}}_\mathbf{s}\right) = V\left(\frac{1}{R_t} - \frac{1}{R_s}\right)\hat{\mathbf{n}}_\mathbf{t}$$

the local rate of change of the unit tangent reads $\dfrac{\partial \beta}{\partial t} = V\left(\dfrac{1}{R_t} - \dfrac{1}{R_s}\right)$

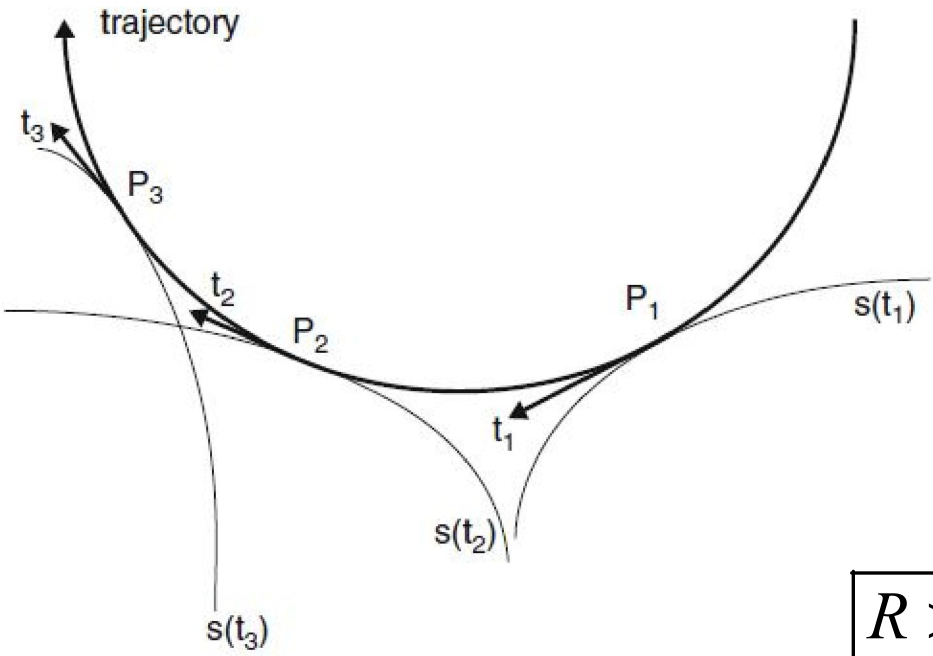


$\beta =$ direction of the wind at each point

$R_t =$ Radius of curvature of trajectory

$R_s =$ Radius of curvature of streamline

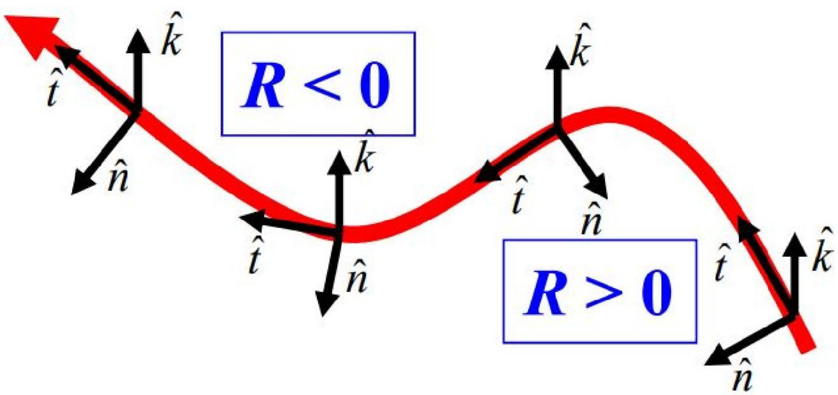
$$\frac{d\beta}{ds} = \frac{1}{R_t} \qquad \frac{\partial \beta}{\partial s} = \frac{1}{R_s}$$



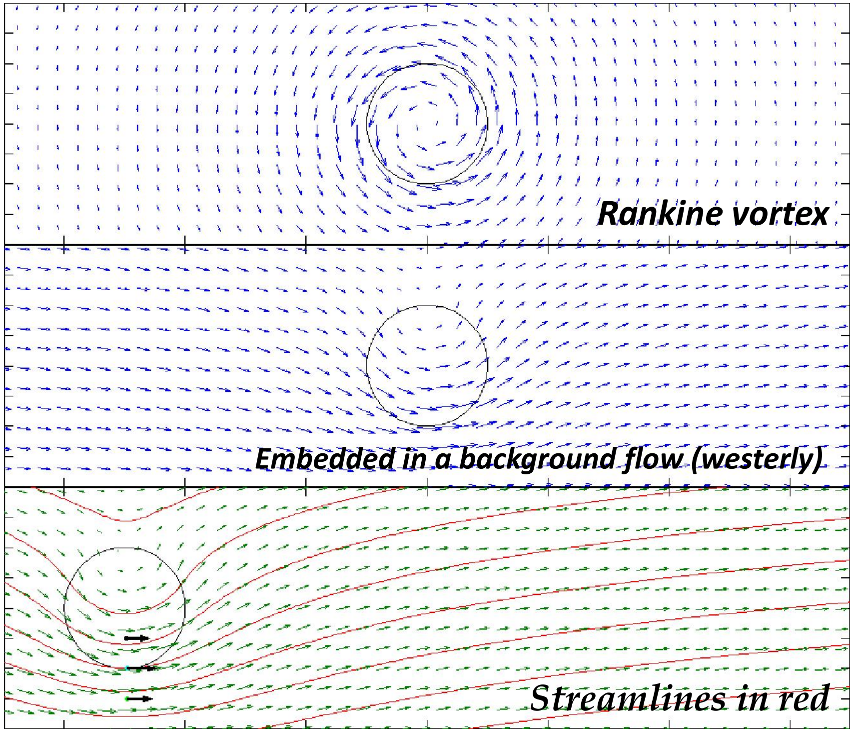
$$\frac{d\beta}{ds} = \frac{1}{R_t} \qquad \frac{\partial\beta}{\partial s} = \frac{1}{R_s}$$

$R > 0 \Leftrightarrow$ Around a low pressure

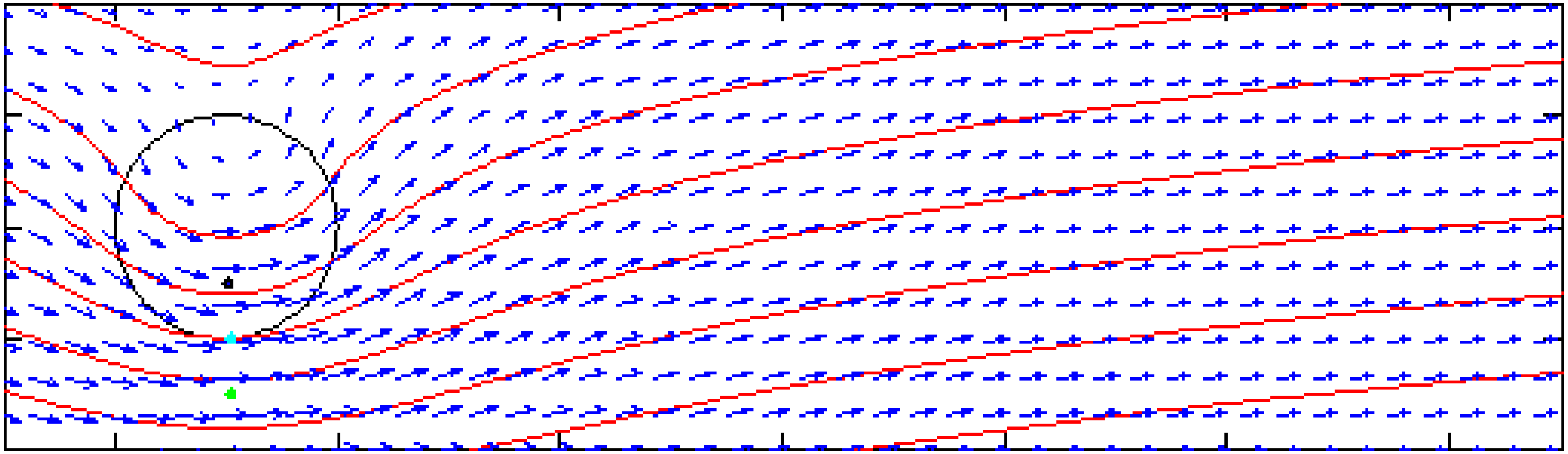
$R < 0 \Leftrightarrow$ Around a high pressure



Trajectory and streamline



- *As the flow evolves in time, the trajectories of three fluid parcels are traced by the continuously lengthening coloured lines.*
- *The black vectors, whose tails coincide with the position of each fluid parcel, show the instantaneous velocity at the location of each parcel.*
- *Notice how the parcels trace out cycloidal patterns that look nothing like the streamlines.*
- *Although at a given instant, streamlines and trajectories are both tangent to the velocity vectors, trajectories may deviate significantly from the streamlines over even a short time interval.*



Kinematics of the pressure field

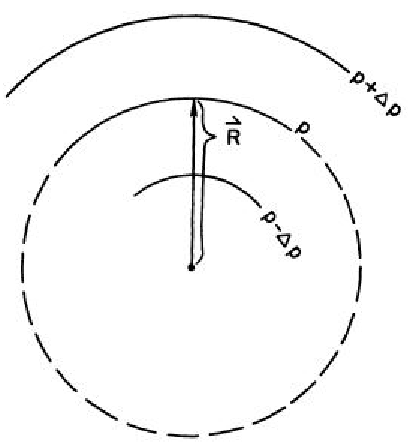
Lines of constant pressure (usually at fixed elevation) are called isobars.

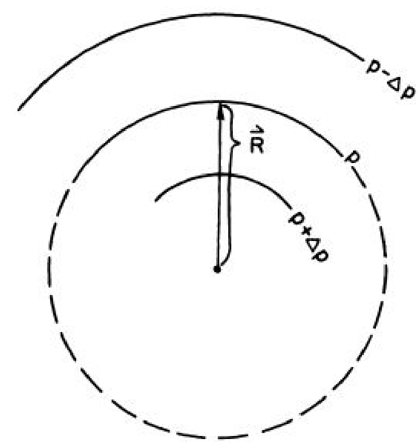
Curved isobar (sea level pressure) $\Leftrightarrow$ Curved height contours (aloft)

Radius of curvature vector ($\vec{\mathbf{R}}$) is measured from the center, radially outward to the isobar.

$\rightarrow$ If pressure increases in the radially outward direction ($\vec{\mathbf{R}} \cdot \nabla p > 0$) then the isobar is positively curved (convex) $\Rightarrow$ pressure is an increasing function of distance from the center $\rightarrow$ R > 0.

$\rightarrow$ If pressure decreases in the radially outward direction ($\vec{\mathbf{R}} \cdot \nabla p < 0$), then the isobar is negatively curved (concave) $\Rightarrow$ pressure is a decreasing function of distance from the center $\rightarrow$ R < 0.





($\vec{\mathbf{R}} \cdot \nabla p > 0$) positively curved $\rightarrow$ R > 0.

($\vec{\mathbf{R}} \cdot \nabla p < 0$) negatively curved $\rightarrow$ R < 0.

Ridge and troughs

A ridge axis (or ridge line) is defined as the locus of maximum negative curvature on a set of adjacent isobars $\rightarrow$ saw-toothed shaped line

A trough axis (or trough line) is defined as the locus of maximum positive curvature on a set of adjacent isobars $\rightarrow$ dashed line

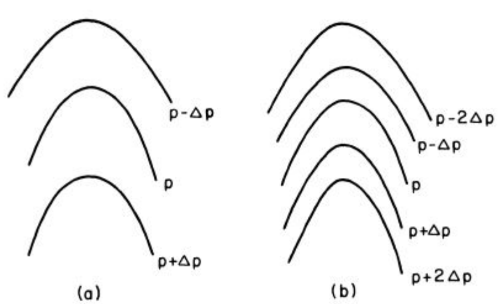
If x-axis is oriented normal to ridge axis, then along the ridge axis, $\frac{\partial p}{\partial x} = 0, \ \frac{\partial^2 p}{\partial x^2} < 0$

Intensity of the ridge is given by $-\frac{\partial^2 p}{\partial x^2}$

If x-axis is oriented normal to trough axis, then along the ridge axis, $\frac{\partial p}{\partial x} = 0, \ \frac{\partial^2 p}{\partial x^2} > 0$

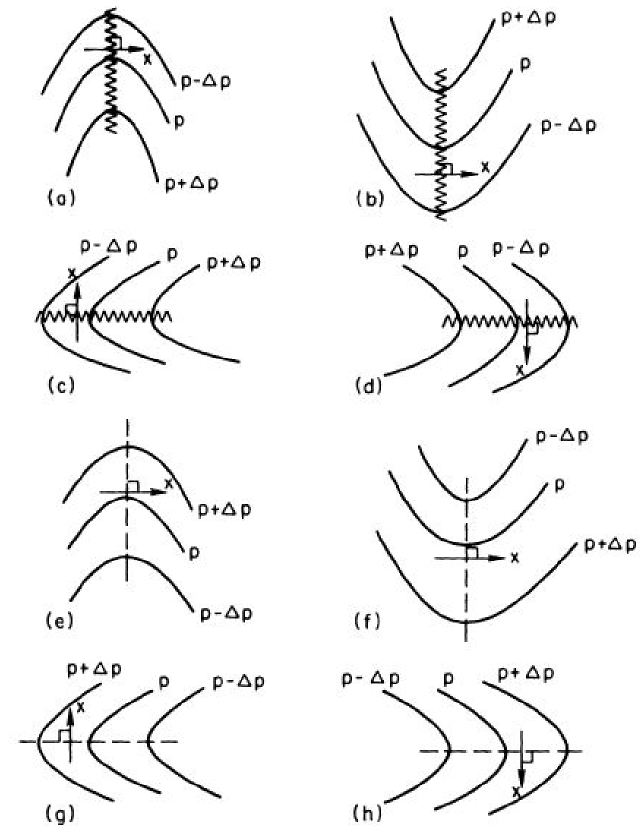
Intensity of the trough is given by $+\frac{\partial^2 p}{\partial x^2}$

Weak ridge

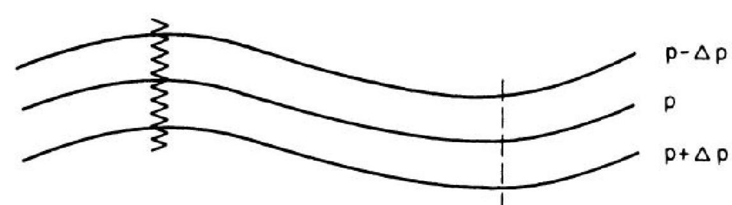


Strong ridge

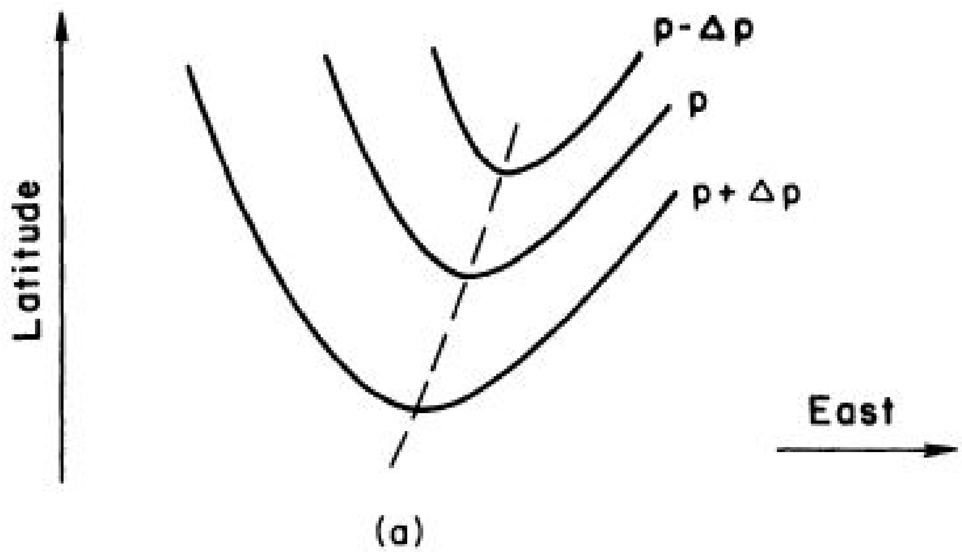
Idealized representations of troughs/ridges in the pressure field



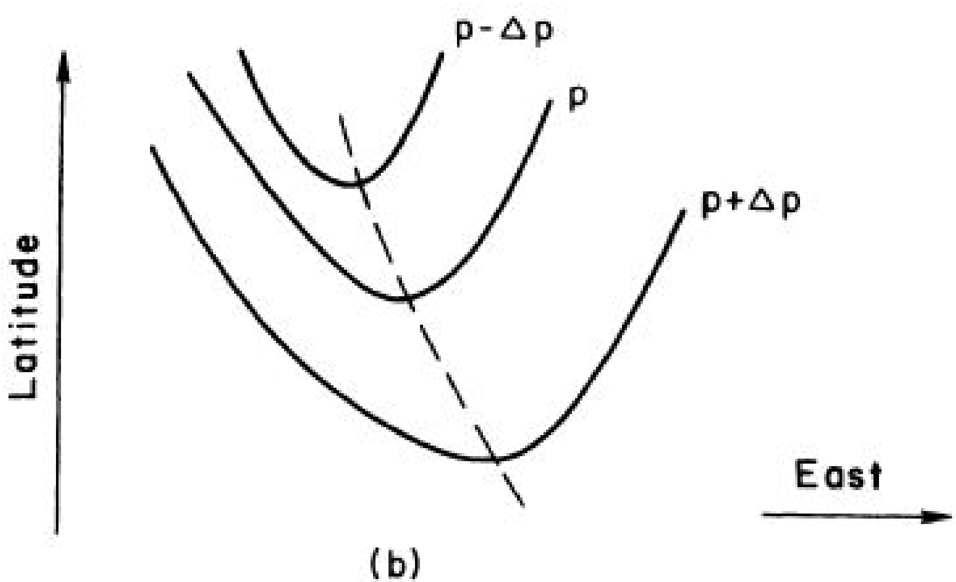
x – axes shown are oriented normal to the ridge/trough axes



Tilted troughs



Positive-tilted trough in the westerlies



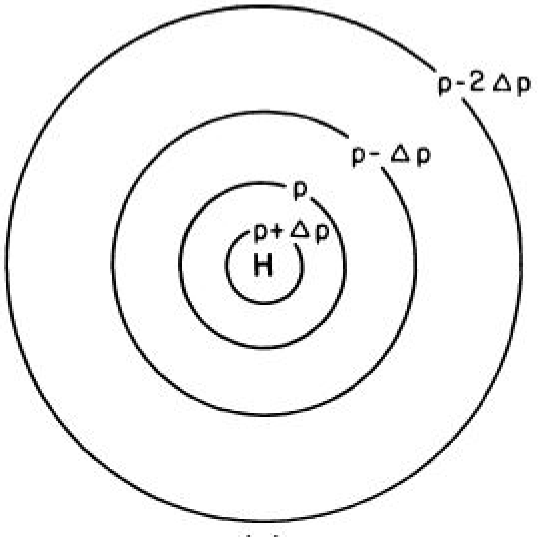
Negative-tilted trough in the westerlies

High and low pressure centers

A high pressure center or high is a local maximum in the pressure field.

At the center of the high: $\nabla_z p = 0$, and $\nabla_z^2 p < 0$

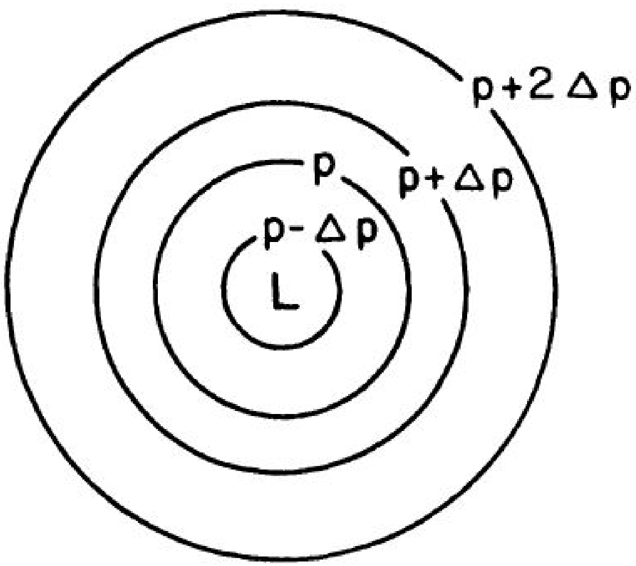
Intensity of the High is given by $-\nabla_z^2 p$



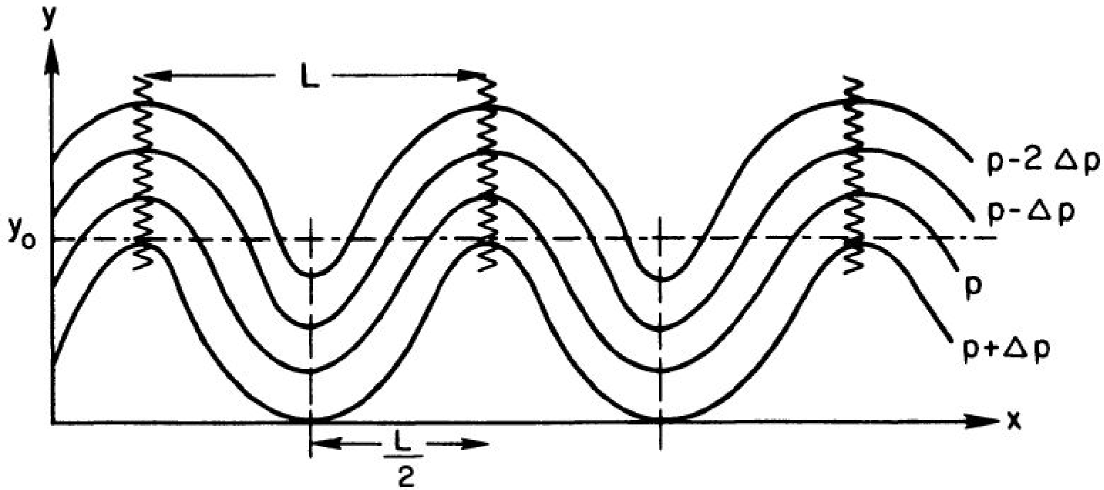
A low pressure center or low is a local minimum in the pressure field.

At the center of the low: $\nabla_z p = 0$, and $\nabla_z^2 p > 0$

Intensity of the Low is given by $+\nabla_z^2 p$



Example of wavetrain

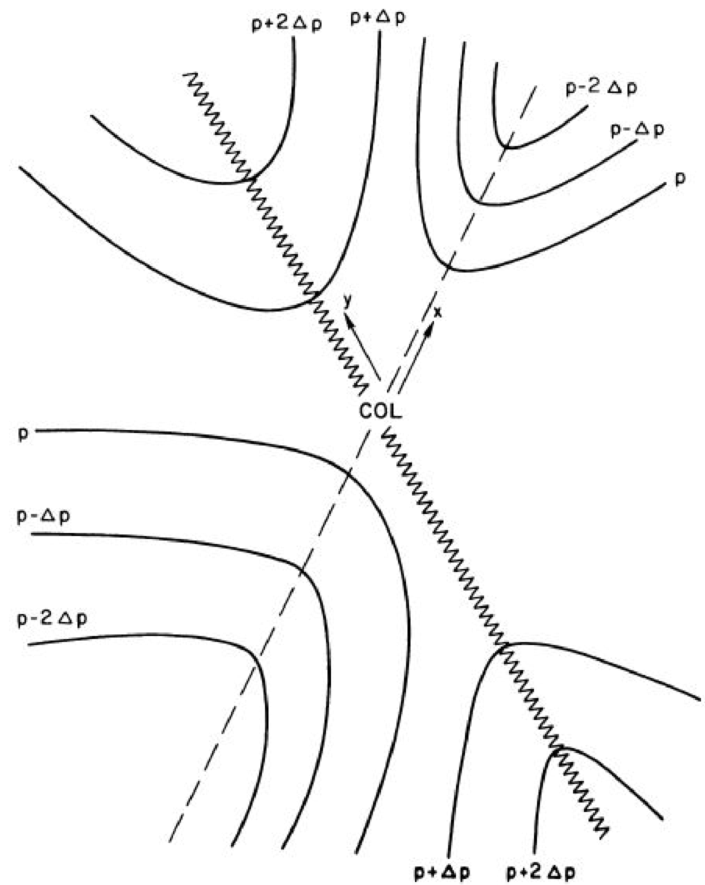


Idealized example of a wavetrain in the westerlies In northern hemisphre

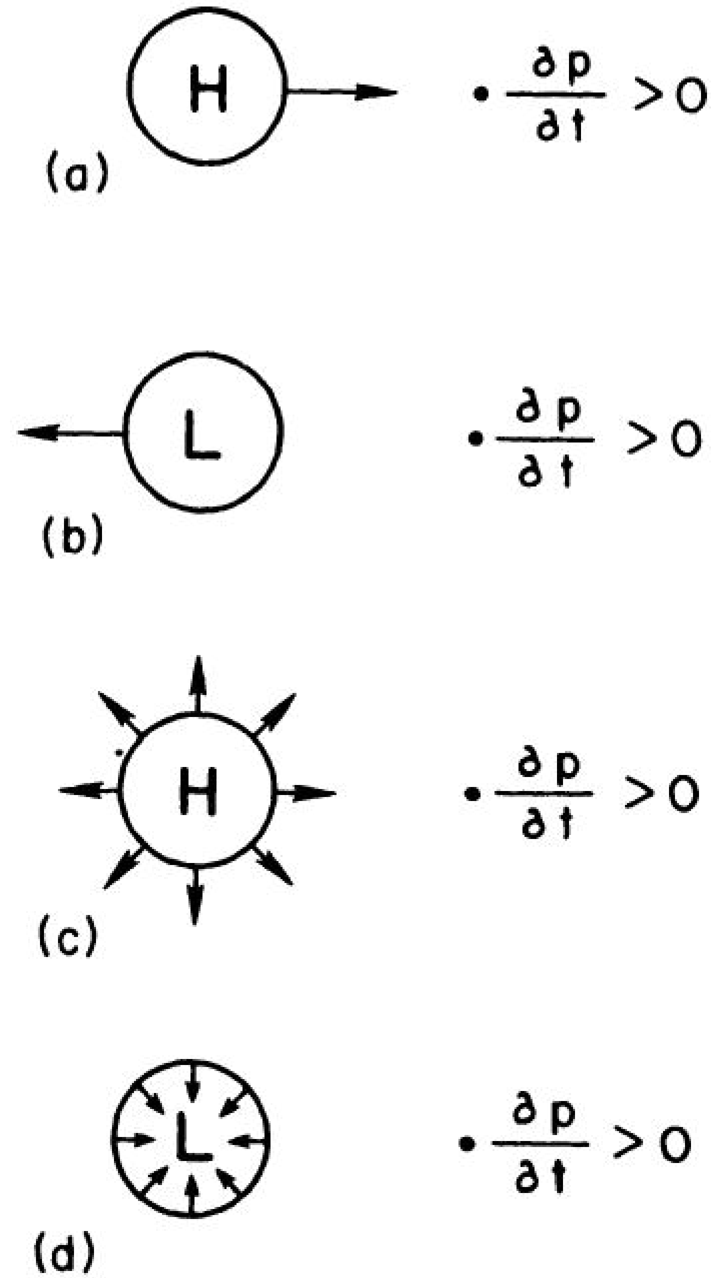
A saddle point (common in horse latitudes) in the pressure field is called a "col"

$\Rightarrow$ COL may be defined as the intersection of at least one trough axis and one ridge axis

$\Rightarrow$ At a COL, $\dfrac{\partial p}{\partial x} = \dfrac{\partial p}{\partial y} = 0$ and $\left(\dfrac{\partial^2 p}{\partial x^2}\right)\left(\dfrac{\partial^2 p}{\partial y^2}\right) < 0$



(a) there is a high nearby propagating towards the observation station
(b) there is a low nearby that is propagating away from the observation station
(c) there is a high nearby that is building
(d) there is a low nearby that is filling
→ Motion of isobars is indicated by arrows.



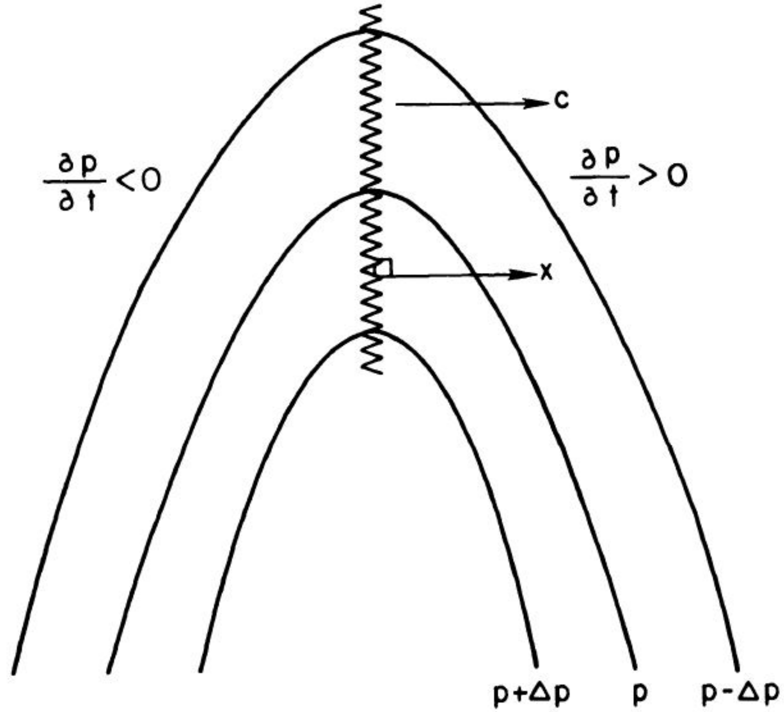


Illustration of the relationship between motion of a ridge and temporal and spatial variations in the pressure field.

If an observer to the right of the ridge (isobars given by solid lines) notes a local pressure rise, and an observer to the left of the ridge notes a pressure fall

→ then the ridge must be moving from left to right.