Scale analysis of atmospheric motions

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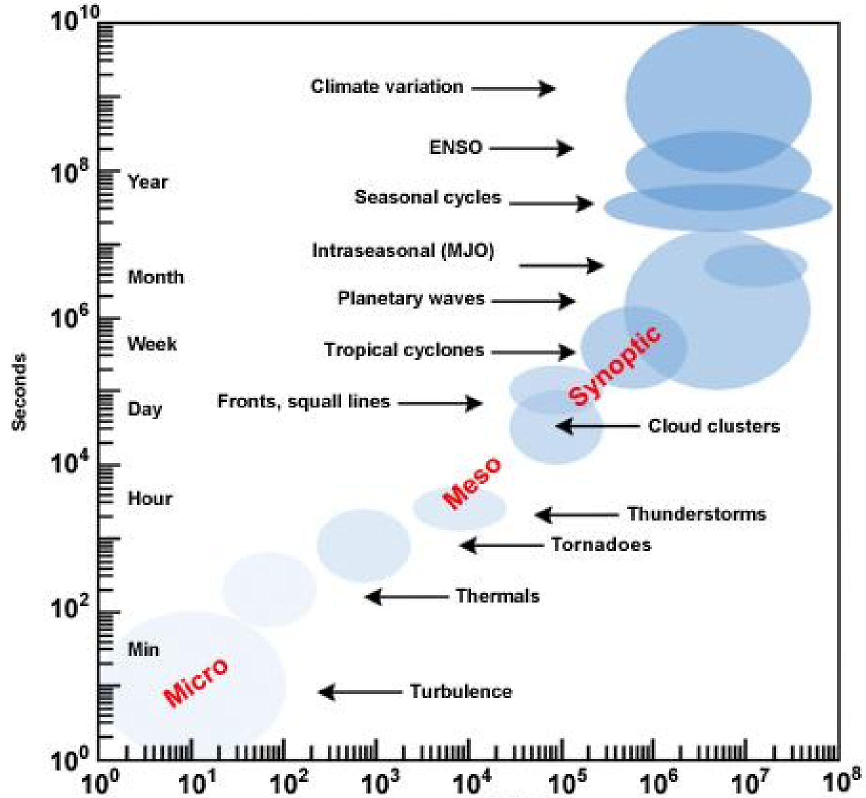
Dynamics of the atmosphere and oceans

2021-22



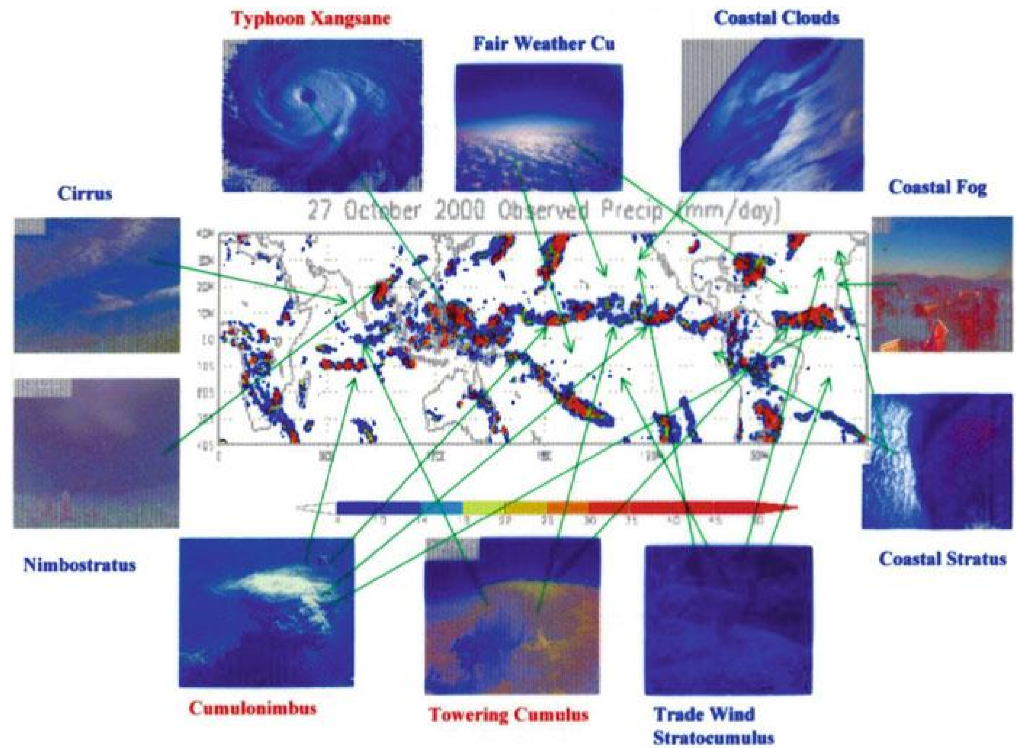
➢ ***Classify weather systems according to their intrinsic or characteristic time and space scales.***

Scale name	Scale dimension	Examples
Molecular scale	≪2 mm	Molecular diffusion, molecular viscosity
Microscale	2 mm–2 km	Eddies, small plumes, car exhaust, cumulus clouds
Mesoscale	2–2000 km	Gravity waves, thunderstorms, tornados, cloud clusters, local winds, urban air pollution
Synoptic scale	500–10 000 km	High- and low-pressure systems, weather fronts, tropical storms, hurricanes, Antarctic ozone hole
Planetary scale	>10 000 km	Global wind systems, Rossby (planetary) waves, stratospheric ozone reduction, global warming

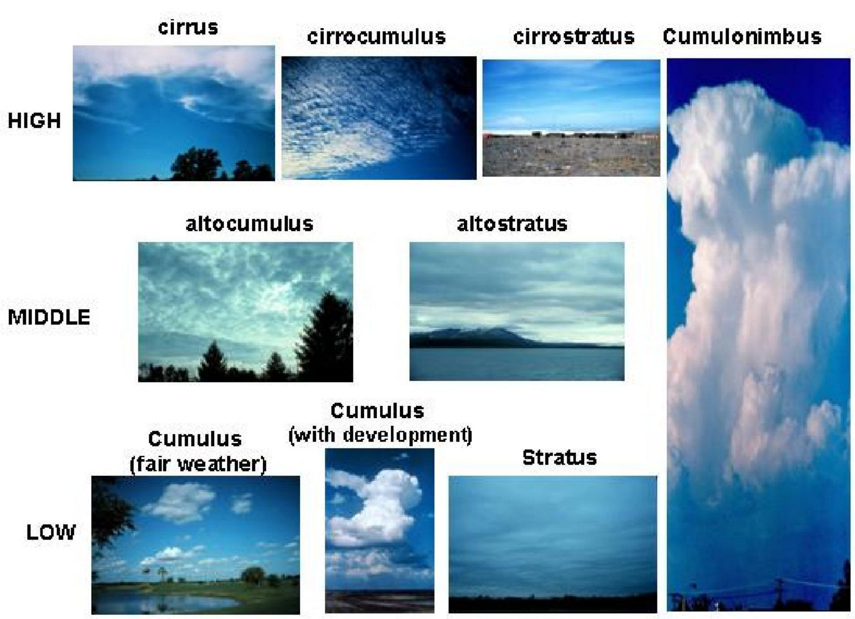
Length, velocity and time scales in the Earth's atmosphere and oceans

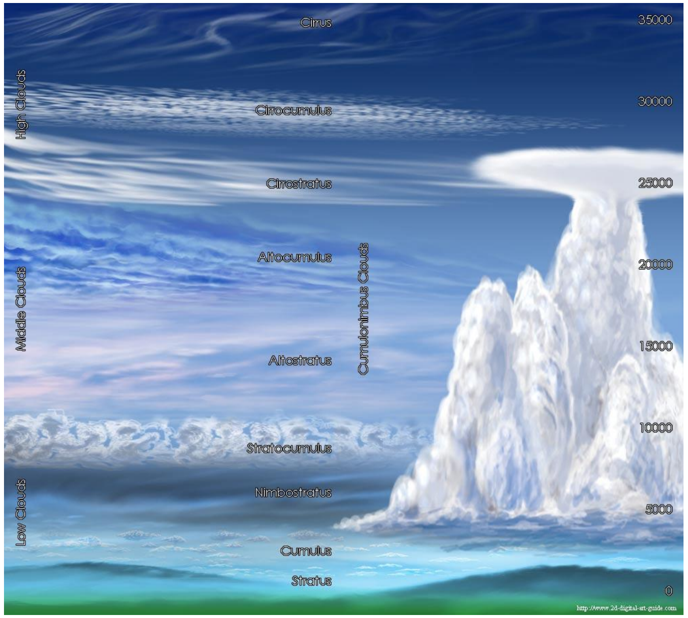
Phenomenon	Length scale L	Velocity scale U	Time scale T
Atmosphere:			
Sea breeze	5–50 km	1–10 m/s	12 h
Mountain waves	10–100 km	1–20 m/s	Days
Weather patterns	100–5000 km	1–50 m/s	Days to weeks
Prevailing winds	Global	5–50 m/s	Seasons to years
Climatic variations	Global	1–50 m/s	Decades and beyond
Ocean:			
Internal waves	1–20 km	0.05–0.5 m/s	Minutes to hours
Coastal upwelling	1–10 km	0.1–1 m/s	Several days
Large eddies, fronts	10–200 km	0.1–1 m/s	Days to weeks
Major currents	50–500 km	0.5–2 m/s	Weeks to seasons
Large-scale gyres	Basin scale	0.01–0.1 m/s	Decades and beyond

Scales of atmospheric motion



On a particular day ...





The basic scale dimensions

Symbol	Variable	Units
U	horizontal velocity	$m\ s^{-1}$
W	vertical velocity	$m\ s^{-1}$
L	Length	m
H	height or depth	m
δP/ρ	horizontal pressure fluctuation weighted by density	$J\ kg^{-1} = m^2\ s^{-2}$, units of geopotential
T = L/U	Time	s

Horizontal Length scale (L):

L can be defined in a few ways

$\Rightarrow$ For wavelike features in the atmosphere it is usually taken to be one-fourth of the total wavelength

$L \approx \dfrac{\lambda}{4}$ Phillips, N.A. (1963): 'Geostrophic motion' Rev. Geophys. 1, 123–175

$\Rightarrow$ Based on the concept of "Rossby radius"

Vertical Length scale (H): is the height of the circulation or disturbance, generally $\rightarrow$ height of the troposphere

Horizontal and vertical motion:

$\Rightarrow$ Horizontal velocity (U): For most atmospheric circulations the u and v components are of similar magnitude, and so we use a single scale parameter, U, to represent both.

$\Rightarrow$ Vertical velocity (W)

δp : Pressure change $\rightarrow$ In the horizontal, this will be the range between the maximum and minimum pressures found moving horizontally across the circulation

In the vertical, it will be the maximum and minimum pressures found moving vertically through the circulation

$$(\delta p)_{\text{horizontal}} << (\delta p)_{\text{vertical}}$$

Time (τ) : For the time scale we use the advective time scale, defined as $\tau_{\text{adv}} = L/U$. This is the time it would take for a parcel of fluid traveling at speed U to travel the distance L

Arbitrary division of space/time

Synoptic $\Rightarrow$ the length scale of the phenomenon $(L) \geq$ 1000 km
or time scale of the phenomenon $(T) >$ 1 day

$\Leftrightarrow$ geostrophic balance
hydrostatic balance

Mesoscale $\Rightarrow$ 10 km $< L <$ 1000 km;
1h $< T <$ 1 day
MCC $\Rightarrow$ T = 10 hr, L = 250 km

$\Leftrightarrow$ small enough to be significantly **out of geostrophic balance**, but large enough that hydrostatic approximation is valid

Convective $\Rightarrow$ $L <$ 10 km
e.g., Turbulence $\Rightarrow$ T = 10 sec, L = 1 m
Thermal $\Rightarrow$ T = 5 min, L = 500 m
Cb $\Rightarrow$ T = 30 min, L = 3 km

$\Leftrightarrow$ neither geostrophic balance nor hydrostatic balance

Question: How the magic number $L \approx$ 1000 km came for synoptic scale motions?

$\Rightarrow$ definition of Rossby radius

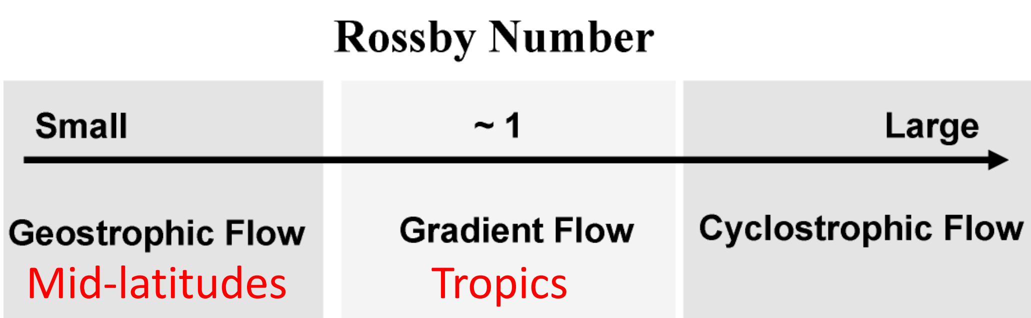
Rossby radius: is the characteristic horizontal length scale at which rotation effects become as important as buoyancy effects

Rossby number – characterizes atmospheric flow regimes

- **When the Rossby number is large (such as in the tropics and at lower latitudes), the effects of planetary rotation are unimportant and can be neglected.**
- **When the Rossby number is small (Ro << 1), the effects of planetary rotation are large.**

$$Ro = \frac{U}{fL}$$

Balanced Flows



Despite the apparent complexity of atmospheric motion systems as depicted on synoptic weather charts, the pressure (or geopotential height) and velocity distributions in meteorological disturbances are actually related by rather simple approximate force balances.

Horizontal momentum equation

$$\frac{d\vec{\mathbf{V}}_{\mathbf{H}}}{dt} + f\,\hat{\mathbf{k}} \times \vec{\mathbf{V}}_{\mathbf{H}} = -\vec{\nabla}_p \Phi$$

Inertial acceleration + Coriolis acceleration = Pressure-gradient force

$$Ro = \text{Rossby number} \Rightarrow \frac{\text{Inertial acceleration}}{\text{Coriolis acceleration.}}$$

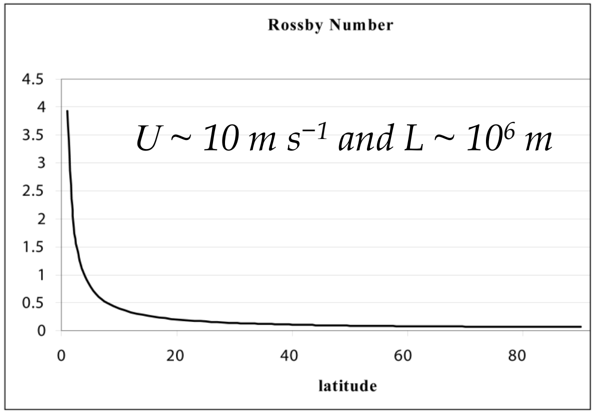
$$\mathbf{Ro} \equiv \frac{d\vec{\mathbf{V}}_{\mathbf{H}}}{dt} \Big/ f\hat{\mathbf{k}} \times \vec{\mathbf{V}}_{\mathbf{H}}$$

Rossby number – characterizes atmospheric flow regimes

- A small Rossby number ($Ro << 1$) implies that the inertial terms are unimportant, and that the pressure gradient force balances the Coriolis force (geostrophic balance). A large Rossby number ($R_0 >> 1$) implies that the Coriolis term is unimportant, and that the inertial terms and the pressure gradient term balance (cyclostrophic balance).
- For synoptic scale motions we typically use values of velocity scale $U \sim 10\ m\ s^{-1}$ and length scale $L \sim 10^6\ m$. The graph below shows the Rossby number as a function of latitude for these values of U and L.
- Poleward of 20° the Rossby number is small enough that the geostrophic wind and actual wind are fairly close. Equatorward of 20° the Rossby number is no longer small, and the geostrophic and actual wind can differ greatly, especially as the Equator is approached.
- The comparatively large values of Rossby number in the Tropics vs. the mid-latitudes means that on the synoptic scale quasi-geostrophic theory isn't very useful in explaining the dynamics of synoptic-scale tropical circulations.

On the planetary scale, where $L \sim 10^7$ m, the Rossby number does remain small through most of the Tropics. Therefore, quasi-geostrophic theory may be carefully applied to planetary-scale circulations such as the monsoon, the Walker circulation, etc.

$$Ro = \frac{U}{fL}$$



Scale analysis of atmospheric motions (small Ro)

Not all of the terms in the momentum equations are significant. If a term is much smaller than the others, then it is reasonable to ignore it under certain circumstances

$$\mathrm{Ro} \approx 0.1$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + 2\Omega \sin\varphi v - 2\Omega w \cos\varphi + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) + \nu\frac{\partial^2 u}{\partial z^2}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial y} - 2\Omega \sin\varphi u + \nu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) + \nu\frac{\partial^2 v}{\partial z^2}$$

$\frac{U^2}{L}$	$\frac{U^2}{L}$	$\frac{UW}{H}$	$\frac{\delta p_H}{\rho L}$	$f_0 U$	$f_0 W$	$\nu\frac{U}{L^2}$	$\nu\frac{U}{H^2}$
10^{-4}	10^{-4}	10^{-5}	10^{-3}	10^{-3}	10^{-6}	10^{-16}	10^{-12}

$U = 10$ m s^{-1}

$W = 1$ cm s^{-1}

$L = 1000$ km

$H = 10$ km

$T \equiv 10^5$ sec

$\mathrm{Ro} = \frac{U}{f_0 L} \approx 0.1$

$\frac{H}{L} << 1 \rightarrow$ hydrostatic balance

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + 2\Omega \cos\varphi u - g + \nu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) + \nu\frac{\partial^2 w}{\partial z^2}$$

$\frac{UW}{L}$	$\frac{UW}{L}$	$\frac{W^2}{H}$	$\frac{\delta p_V}{\rho H}$	$f_0 U$		$\nu\frac{W}{L^2}$	$\nu\frac{W}{H^2}$
10^{-7}	10^{-7}	10^{-8}	10	10^{-3}	10	10^{-19}	10^{-15}

$\delta p_H = 10$ hPa

$\delta p_V = 1000$ hPa

$\rho = 1$ kg m^{-3}

$f_0 \approx 10^{-4}$ s^{-1}

$\nu = 10^{-5}$ m^2 s^{-1}

When the Rossby number is much less than unity (Ro << 1), then the acceleration (inertial) term can be ignored, and the only two terms left are the PG term and Coriolis term, which must be nearly in balance

⇔ synoptic scale motions tend to approach "geostrophic balance" and in "hydrostatic balance"

Synoptic scale vorticity dynamics (small Ro)– Scale analysis

$$\frac{\partial \zeta}{\partial t} + \vec{\mathbf{V}}_H \cdot \nabla(\zeta + f) + w\frac{\partial \zeta}{\partial z} = -(\zeta + f)\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) - \left(\frac{\partial w}{\partial x}\frac{\partial v}{\partial z} - \frac{\partial w}{\partial y}\frac{\partial u}{\partial z}\right) + \frac{1}{\rho^2}\left(\frac{\partial \rho}{\partial x}\frac{\partial p}{\partial y} - \frac{\partial \rho}{\partial y}\frac{\partial p}{\partial x}\right)$$

$\frac{U^2}{L^2}$	$\frac{WU}{HL}$	$\frac{f_0 U}{L}$	$\frac{WU}{HL}$	$\frac{\delta p \delta \rho}{\rho^2 L^2}$
10^{-10}	10^{-11}	10^{-9}	10^{-11}	10^{-11}

For midlatitude synoptic-scale systems, the relative vorticity is often small compared to the planetary vorticity [$\zeta (10^{-5}) \ll f\ (10^{-4}$ at 45^oN)].

On the synoptic scale, the total absolute vorticity advection can be approximated by the horizontal advection

$$\frac{\partial \zeta}{\partial t} + \vec{\mathbf{V}}_H \cdot \nabla(\zeta + f) = -f\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) \approx f\frac{\partial w}{\partial z}$$

synoptic scale vorticity processes in mid-latitudes is governed by horizontal convergence/divergence and associated vertical motion

For synoptic-scale flows, the tilting term, solenoidal term, and vertical advection of vorticity can be neglected, f plays a dominant role in the divergence term. For intense ($\zeta \approx f$) cyclonic storms,

$$\frac{\partial \zeta}{\partial t} + \vec{\mathbf{V}}_H \cdot \nabla(\zeta + f) = -(\zeta + f)\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)$$

$U \sim 10$ m s^{-1}
$W \sim 1$ cm s^{-1}
$L \sim 10^6$ m
$H \sim 10^4$ m
$\delta p \sim 10$ hPa
$\rho \sim 1$ kg m^{-3}
$\delta\rho/\rho \sim 10^{-2}$
$L/U \sim 10^5$ s
$f_0 \sim 10^{-4}$ s^{-1}
$\beta \sim 10^{-11}$ m^{-1} s^{-1}

$$\zeta \approx \frac{U}{L} \approx 10^{-5}\ \text{s}^{-1} < f$$

(small Ro)– Scale analysis of thermodynamic energy equation

$$\frac{ds}{dt} \equiv c_p \frac{d\ln\theta}{dt} = \frac{\dot{Q}}{T}$$ ⇔ This can be linearized by letting $\theta(x,y,z,t) = \theta_o(z) + \theta'(x,y,z,t)$

Assuming $\left|\frac{\theta'}{\theta_0}\right| << 1$, $\left|\frac{d\theta'}{dz}\right| << \left|\frac{d\theta_0}{dz}\right|$ and $\ln\theta = \ln\left[\theta_0\left(1+\frac{\theta'}{\theta_0}\right)\right] \approx \ln\theta_0 + \frac{\theta'}{\theta_0}$

$$c_p \frac{d\ln\theta}{dt} = \frac{\dot{Q}}{T} \Rightarrow c_p \frac{d}{dt}\left\{\ln\theta_0 + \frac{\theta'}{\theta_0}\right\} = \cancel{c_p \frac{d\ln\theta_0}{dt}} + \frac{c_p}{\theta_0}\frac{d\theta'}{dt} \Leftrightarrow \boxed{\frac{c_p}{\theta_0}\frac{d\theta'}{dt} = \frac{\dot{Q}}{T}}$$

$$\frac{1}{\theta_0}\left(\frac{\partial\theta'}{\partial t} + \vec{\mathbf{V}}_{\mathbf{H}} \cdot \nabla\theta'\right) + w\left(\frac{1}{\theta_0}\frac{d\theta_0}{dz}\right) = \frac{\dot{Q}}{c_p T}$$

The above equation may also be written as $\frac{\partial\theta'}{\partial t} + \vec{\mathbf{V}}_{\mathbf{H}} \cdot \nabla\theta' + \left(\frac{N^2\theta_0}{g}\right)w = \frac{\theta_0}{c_p T}\dot{Q}$ where $N^2 \approx \frac{g}{\theta_0}\frac{d\theta_0}{dz}$

For the mid-latitude scales, $U = 10\ \text{m s}^{-1}$, $L = 10^6\ \text{m}$, $\theta_0 \approx 300\ \text{K}$, $\theta' \approx 4\ \text{K}$, $N \approx 0.01\ \text{s}^{-1}$, $\dot{Q}/c_p < 1\ \text{K day}^{-1}$

$$\frac{\partial\theta'}{\partial t} + \vec{\mathbf{V}}_{\mathbf{H}} \cdot \nabla\theta' + \left(\frac{N^2\theta_0}{g}\right)w = 0$$

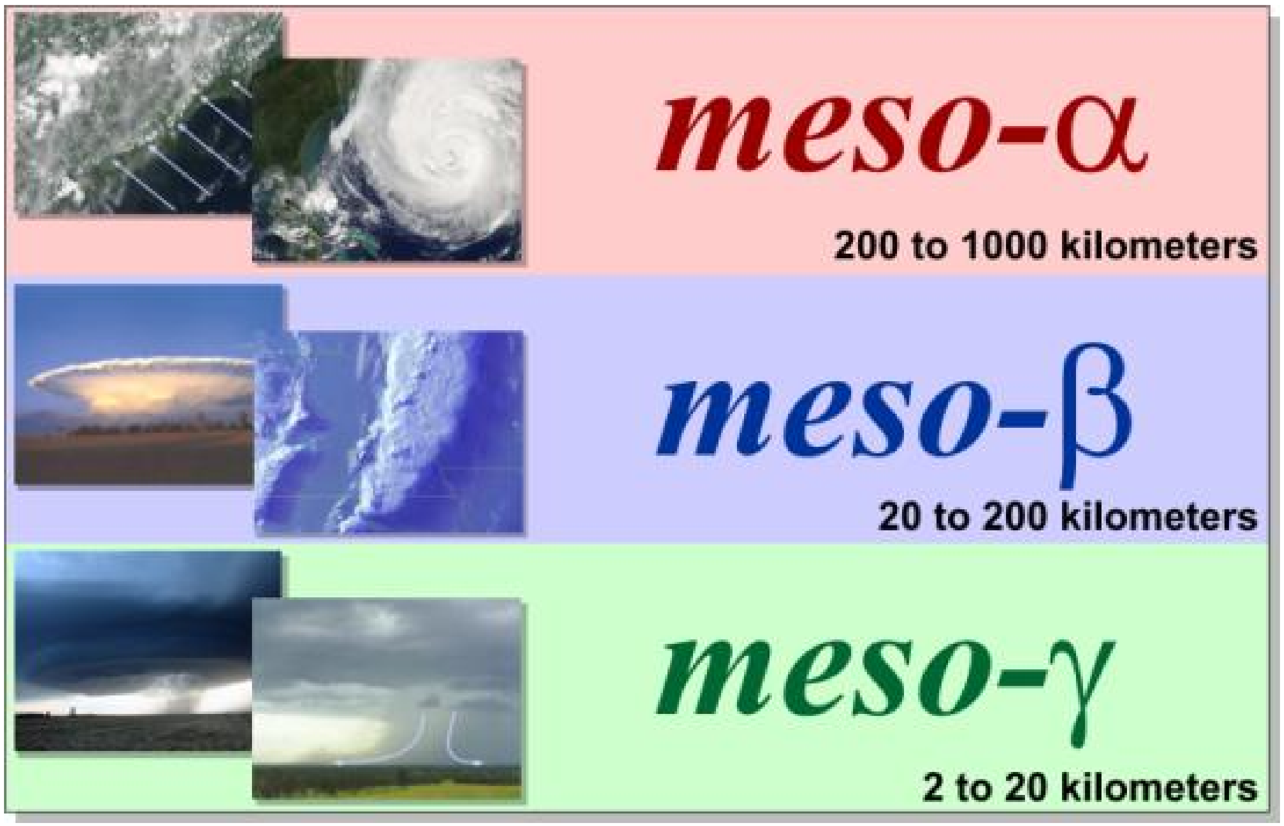
Alternatively,

$$\frac{\partial T}{\partial t} + \vec{\mathbf{V}}_{\mathbf{H}} \cdot \nabla T + \left(\Gamma_d - \Gamma\right)w = 0$$

⇒ in the absence of strong diabatic heating, the rate of change in θ' is equal to the adiabatic heating or cooling due to vertical motion in the statically stable basic state

Synoptic scale motions in mid-latitudes is largely governed by horizontal temperature advection and static stability, less influenced by diabatic heating.

Mesoscale motions (Ro greater than or equal to 1)



Jet streaks
Tropical cyclone

meso-β features are sea and lake-breeze circulations, MCS

Rotating thunderstorms, Tornadoes, dust devil

Mesoscale motions

Moderate Ro flows

$\mathrm{Ro} \approx 1$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + 2\Omega\sin\varphi v$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial y} - 2\Omega\sin\varphi u$$

$\frac{U^2}{L}$	$\frac{U^2}{L}$	$\frac{UW}{H}$	$\frac{\delta p_H}{\rho L}$	$f_0 U$
10^{-3}	10^{-3}	10^{-3}	10^{-3}	10^{-3}

All terms in the equation are of the same magnitude

We no longer have geostrophy!

$U = 10 \text{ m s}^{-1}$

$W = 1 \text{ m s}^{-1} \uparrow$

$L = 100 \text{ km} = 10^5 \text{ m} \downarrow$

$H = 10 \text{ km} = 10^4 \text{ m}$

$T = 10^4 \text{ s} \downarrow$ $(< 1 \text{ day})$

$\downarrow\uparrow$ From Synoptic Scale

e.g., MCC $\Rightarrow$ T = 10 hr, L = 250 km

meso-α (200 – 1000 km, 6 hours – 2 days)

meso-β (20 – 200 km, 30 min – 6 hours)

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g$$

$\frac{UW}{L}$	$\frac{UW}{L}$	$\frac{W^2}{H}$	$\frac{\delta p_V}{\rho H}$	
10^{-4}	10^{-4}	10^{-4}	10	10

$\Rightarrow$ hydrostatic balance is still a good approximation for mesoscale flows $\rightarrow$ The vertical motion is much smaller than the horizontal motion $(W < U)$.

$\delta p_H = 1 \text{ hPa}$

$\approx 1 \text{ hPa/100 km}$

$\approx 10 \text{ hPa/1000 km}$

$\delta p_V = 1000 \text{ hPa}$

$\rho = 1 \text{ kg m}^{-3}$

$f_0|_{45^o\text{N}} \approx 10^{-4} \text{ s}^{-1}$

ageostrophy is significant

quasi 2D = $|w| << |u, v|$

$\Rightarrow$ f is not negligible

$\Rightarrow$ nearly hydrostatic

$W < U, \quad \frac{H}{L} << 1$

Mesoscale motions

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + 2\Omega\sin\varphi v$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial y} - 2\Omega\sin\varphi u$$

$$10^{-3} \quad 10^{-3} \quad 10^{-3} \quad 10^{-3} \quad 10^{-3}$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g$$

$$10^{-4} \quad 10^{-4} \quad 10^{-4} \quad \boxed{10} \quad \boxed{10}$$

$U = 10 \text{ m s}^{-1}$
$W = 1 \text{ m s}^{-1} \uparrow$
$L = 100 \text{ km} = 10^5 \text{ m} \downarrow$
$H = 10 \text{ km} = 10^4 \text{ m}$
$T = 10^4 \text{ s} \downarrow$ (< 1 day)

$\delta p_H = 1 \text{ hPa}$
$\delta p_V = 1000 \text{ hPa}$
$\rho = 1 \text{ kg m}^{-3}$
$f \approx 10^{-4} \text{ s}^{-1}$

(1) Local time tendency terms $\frac{\partial u}{\partial t}, \frac{\partial v}{\partial t}, \frac{\partial w}{\partial t}$ can be neglected $\Leftrightarrow$ if the movement of the mesoscale system is much less than the advecting wind speed ($c_{system} << U$). Such a system is said to be steady state if $c_{system} = 0$ or quasi-steady if $c_{system} \neq$ but still $c_{system} << U$.

(2) If horizontal scale is sufficiently large $\Rightarrow$ hydrostatic equation is valid.

(3) If horizontal scale is sufficiently small $\Rightarrow$ the Coriolis term is small relative to the advective and pressure gradient forces (Ro >>1)

Mesoscale vorticity dynamics - Scale analysis

$$\frac{\partial \zeta}{\partial t} + \vec{\mathbf{V}}_{\mathbf{H}} \cdot \nabla(\zeta + f) + w\frac{\partial \zeta}{\partial z} = -(\varsigma + f)\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) - \left(\frac{\partial w}{\partial x}\frac{\partial v}{\partial z} - \frac{\partial w}{\partial y}\frac{\partial u}{\partial z}\right) + \frac{1}{\rho^2}\left(\frac{\partial \rho}{\partial x}\frac{\partial p}{\partial y} - \frac{\partial \rho}{\partial y}\frac{\partial p}{\partial x}\right)$$

$\frac{U^2}{L^2}$	$\frac{WU}{HL}$	$\frac{\zeta_0 U}{L}$	$\frac{WU}{HL}$	$\frac{\delta p \delta \rho}{\rho^2 L^2}$	
10^{-8}	10^{-8}	10^{-8} $(\zeta \approx f)$	10^{-8}	10^{-8}	$\Leftrightarrow$ at 45^oN
10^{-8}	10^{-8}	10^{-8} $(\zeta >> f)$	10^{-8}	10^{-8}	$\Leftrightarrow$ Tropics

Remember:

For mesoscale flows, the tilting term, solenoidal term, and vertical advection of vorticity cannot be neglected

$U = 10$ m s^{-1}

$W = 1$ m s^{-1} ↑

$L = 100$ km $= 10^5$ m ↓

$H = 10$ km $= 10^4$ m

$T = 10^4$ s ↓ ($<$ 1 day)

$\delta p_H = 1$ hPa $= 10$ hPa/1000 km

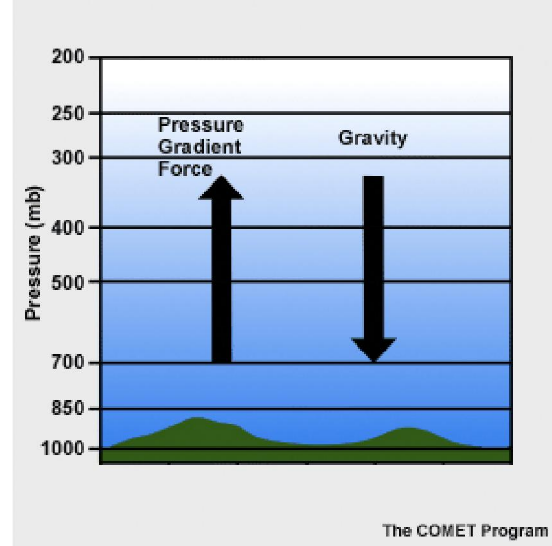
$\delta p_V = 1000$ hPa

$\rho = 1$ kg m^{-3}

$f_0|_{45^o N} \approx 10^{-4}$ s^{-1}

$\zeta_o \approx \frac{U}{L} \approx 10^{-4}$ s^{-1}

Mesoscale-alpha (a)	200 - 2000 km	6 hrs - 2 days	Jet stream, small hurricanes, weak anticyclones
Mesoscale-beta (b)	20 - 200 km	30 mins - 6 hrs	Local wind fields, mountain winds, land/sea breeze, mesoscale convective complexes (MCCs), large thunderstorms
Mesoscale-gamma (c)	2 - 20 km	3 - 30 mins	Most thunderstorms, large cumulus, extremely large tornadoes



One of the keys to mesoscale processes is the role of non-hydrostatic processes.

On a synoptic scale and even upper meso-α, the atmosphere is very nearly in hydrostatic equilibrium.

Consequently, synoptic-scale parcels of air rise and fall very slowly relative to their horizontal motions. However, this is not true for the lower meso-β and especially for meso-γ.

$\Rightarrow$ vertical velocities, driven by processes including buoyancy and topographic effects, can approach or even exceed horizontal velocities (over short distances).

As a result, mesoscale meteorology is frequently determined by non-hydrostatic processes.

Convective/storm scale

$\mathrm{Ro} \approx 10$

High Ro flows

Meso-γ (2 – 20 km)

non-hydrostatic

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + 2\Omega\sin\varphi v - 2\Omega w\cos\varphi$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial y} - 2\Omega\sin\varphi u$$

$\frac{U^2}{L}$	$\frac{U^2}{L}$	$\frac{UW}{H}$	$\frac{\delta p_H}{\rho L}$	$f_0 U$	$f_0 W$
10^{-2}	10^{-2}	10^{-2}	10^{-2}	10^{-3}	10^{-3}

$U = 10$ m s^{-1}; $W = 10$ m s^{-1} $\Uparrow$

$L = 10$ km $= 10^4$ m $\Downarrow$

$H = 10$ km $= 10^4$ m

$T = 10^3$ s $\Downarrow$; $\delta p_H = 1$ hPa $\downarrow$

$\rho = 1$ kg m^{-3}; $f_0 \approx 10^{-4}$ s^{-1}

f can be neglected when studying cumulus convection that lasts for an hour or so. The acceleration term is as important as the PGF.

Three-dimensional ($U \approx W$)

non-hydrostatic

ageostrophic

Coriolis effect is negligible

Following Boussinesq approximation

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = -\frac{1}{\bar{\rho}}\frac{\partial p'}{\partial z} + \frac{\theta'}{\theta_0}g$$

$\frac{UW}{L}$	$\frac{UW}{L}$	$\frac{W^2}{H}$	$\frac{\delta p'}{\bar{\rho}H}$	$\frac{\Delta\theta}{\theta_0}$
10^{-2}	10^{-2}	10^{-2}	10^{-2}	10^{-2}

$\Rightarrow$ Clearly, the vertical acceleration term is now important.

Thus, the hydrostatic approximation is no longer applicable

$\delta p' = 1$ hPa; $\Delta\theta = 10$ K;

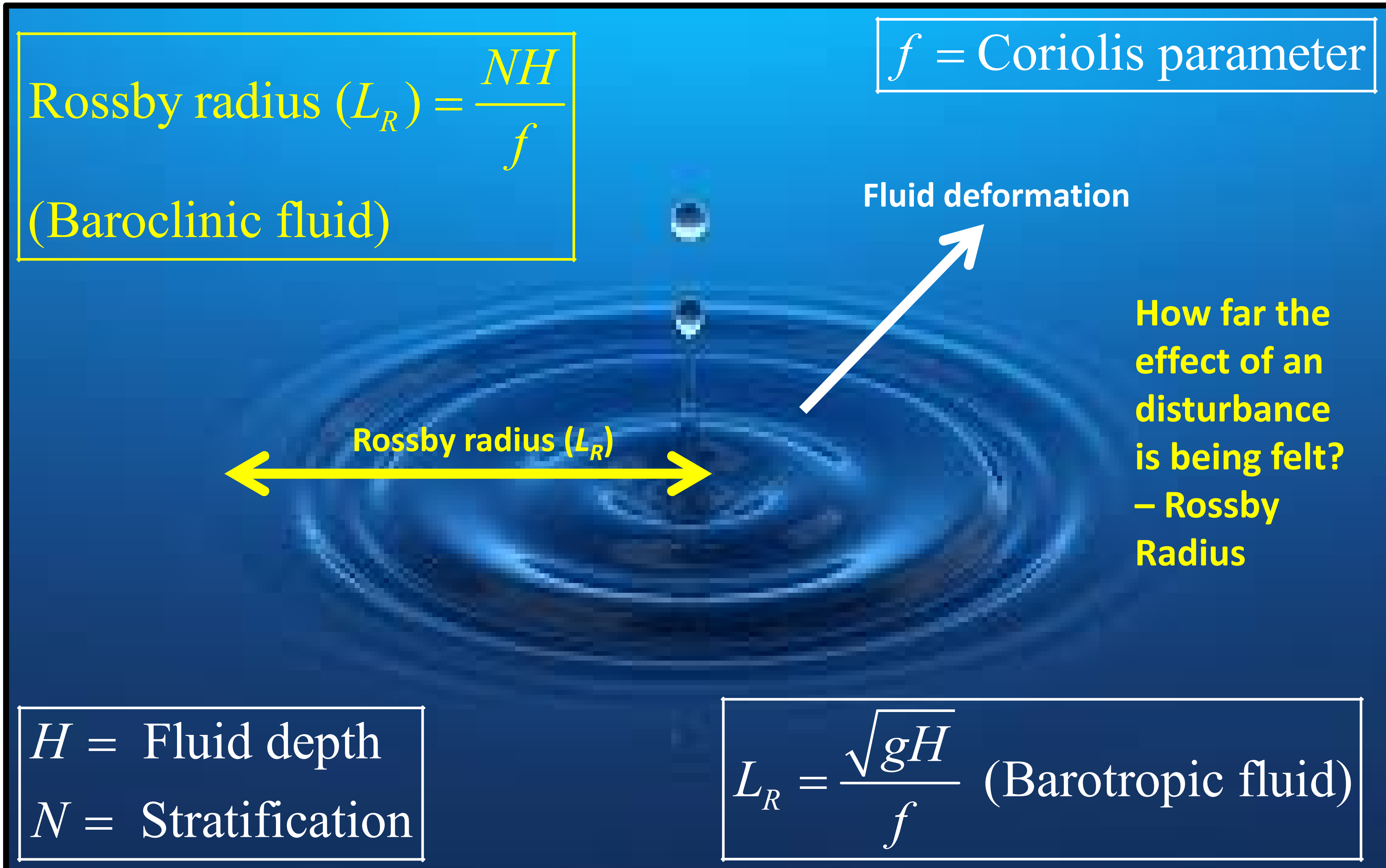
$\theta_0 = 300$ K

$\bar{\rho} \approx 0.5$ kg m^{-3} (vertical mean)

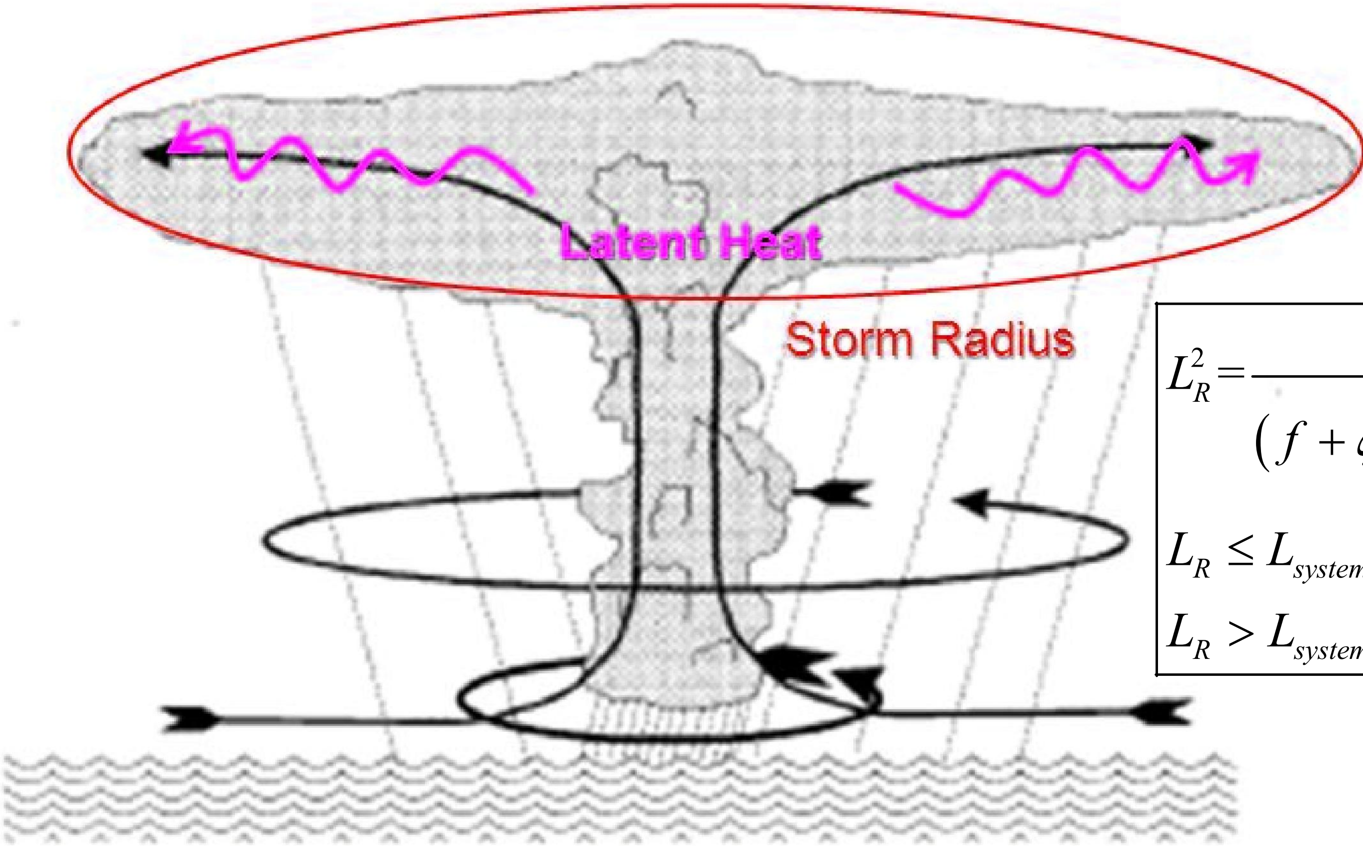
Thermal $\Rightarrow$ T = 5 min, L = 500 m

Cb $\Rightarrow$ T = 30 min, L = 3-20 km

Rossby radius of deformation (L_R)



Systems Persists



$$L_R^2 = \frac{N^2 H^2}{(f+\zeta)\left(f + \frac{2v_{\tan}}{R}\right)}$$

$$L_R \le L_{system} \Rightarrow \text{ system survives}$$

$$L_R > L_{system} \Rightarrow \text{ system dissipates}$$

What is the Rossby Radius of Deformation?

- Distance at which energy disperses by atmospheric waves from the center of a circulation
- If this distance exceeds the storm radius, the energy disperses too far away and the system tends to dissipate.
- If this distance is contained within the system radius, the storm will persist.

$$\text{Rossby radius } (L_R) = \frac{NH}{f}$$

The practical application of L_R is to evaluate whether a pressure or height perturbation feature is dynamically "large" or "small" $\Leftrightarrow$ whether it persists or decays

$\Rightarrow$ If it is dynamically "large", it will retain its perturbation characteristics for a considerable period and winds will come into balance with the mass field (wind adjustment).

$\Rightarrow$ If dynamically "small", the feature will decay and the height field will adjust to the remnants of its wind field (mass adjustment).

Perturbations from Balance

For stable balance, i.e. stability restores balance, perturbations initiate oscillations that result in waves

For unstable balance $\Rightarrow$ perturbations produce a growing disturbance

Perturbations from Hydrostatic Balance: Oscillation frequency N

$\Rightarrow$ Perturbations from stable balance: Gravity or Buoyancy waves –

$$\text{Horizontal phase speed is } c_{gravity} = \frac{L_z}{2\pi}\sqrt{\frac{g}{\theta_0}\frac{\partial\theta}{\partial z}} \quad \left\{\text{Time period } T = \frac{2\pi}{N}\right\}$$

$\Rightarrow$ From unstable balance (buoyancy > gravity) lead to: convection

Perturbations from Geostrophic Balance:

$\Rightarrow$ Stability produces balance: oscillation frequency is f

$$\text{Wave speed is } c_{inertial} = \frac{f}{k} = \frac{f}{2\pi/L_H} \quad \left\{\text{Time period } T = \frac{2\pi}{f}\right\}$$

$\Rightarrow$ Unstable balance produces: Inertial instability

If both hydrostatic and inertial balances occur and the flow is perturbed, what is the result?

$\Rightarrow$ Depends on which adjustment dominates

$\Rightarrow$ Determine dominant adjustment from ratio of gravity wave phase speed to inertial wave phase speed

For stable balance, i.e. stability restores balance, perturbations initiate oscillations that result in waves $\boxed{\dfrac{c_{gravity}}{c_{inertial}} = \dfrac{L_z N}{L_H f}} \Rightarrow \boxed{L_H = \dfrac{NL_z}{f} \approx \dfrac{NH}{f}}$

Scale at which there is equal inertial and gravity wave response

$\Rightarrow$ The definition of Rossby Radius is : $\boxed{L_R = \dfrac{c_{gravity}}{f} = \dfrac{L_z N}{f} \approx \dfrac{NH}{f}}$

$\Rightarrow$ Rossby Radius for axisymmetric vortex having tangential wind $v_{\tan}$ and radius $R \Rightarrow \boxed{L_R = c_{gravity} \Big/ \sqrt{(f+\zeta)\left(f + \dfrac{2v_{\tan}}{R}\right)}}$

What causes perturbations?

When latent heating from convection affects the mass (pressure) field in dynamically large systems, the system will adjust through changes in the rotational part of the wind ($\vec{\mathbf{V}}_\psi$) $\Leftrightarrow$ Wind adjustment

In contrast, for dynamically small systems, adjustment of the mass field to the latent heating will cause divergent circulations ($\vec{\mathbf{V}}_\chi$) that will influence the future evolution of convection ($\Rightarrow$ Mass adjustment $\rightarrow$ making this process difficult to parameterize in numerical modeling!)

$\Rightarrow$ if a tropical disturbance is larger than the Rossby radius, the energy from the gravity waves will be contained within the disturbance, and it will persist.

$\Rightarrow$ if the tropical disturbance is smaller than the Rossby radius, then the energy will be dispersed outside of the radius of the disturbance, and it is more likely to dissipate

Rossby radius $L_R = NH/f$

For a barotropic fluid, $L_R = \frac{\sqrt{gH}}{f}$; For a baroclinic fluid, $L_R = \frac{NH}{f}$

$L, H =$ Horizontal and vertical scales of the imbalance

How far the effect of the disturbance is being felt? – Rossby Radius

Typical values of L_R

Middle-to-high latitudes, $f \approx 10^{-4}\ \text{s}^{-1}$, $H \approx 10$ km, $T_{buoyancy} \approx 10$ min $\Rightarrow L_R \approx 500 - 1000$ km

At 10^0N, $f \approx 2.5\times10^{-5}\ \text{s}^{-1}$, $H \approx 20$ km, $T_{buoyancy} \approx 10$ min $\Rightarrow L_R \approx 8000$ km

At 5^0N, $f \approx 1.3\times10^{-5}\ \text{s}^{-1}$, $H \approx 20$ km, $T_{buoyancy} \approx 10$ min $\Rightarrow L_R \approx 15000$ km

Tropical phenomena are $\rightarrow$ planetary scale
e.g., Monsoon,
Walker circulation

L_R approaches ∞ at the equator as f goes to zero.

Rossby radius of deformation (L_R): is the distance a gravity wave travels during one inertial period

(a) Inertial period (days)

Period	5 °N	10 °N	15 °N	20 °N	30 °N	45 °N
	5.6	2.9	1.9	1.5	1.0	0.7

(b) Atmosphere Rossby radius of deformation L_R

	5°N	10°N	15°N	20°N	30°N	45°N
20 m	1101	553	371	281	192	136
50 m	1742	874	586	444	304	215
100 m	2463	1236	829	627	429	303
400 m	4926	2472	1658	1225	859	607

Inertial period $(T) = \dfrac{2\pi}{f}$

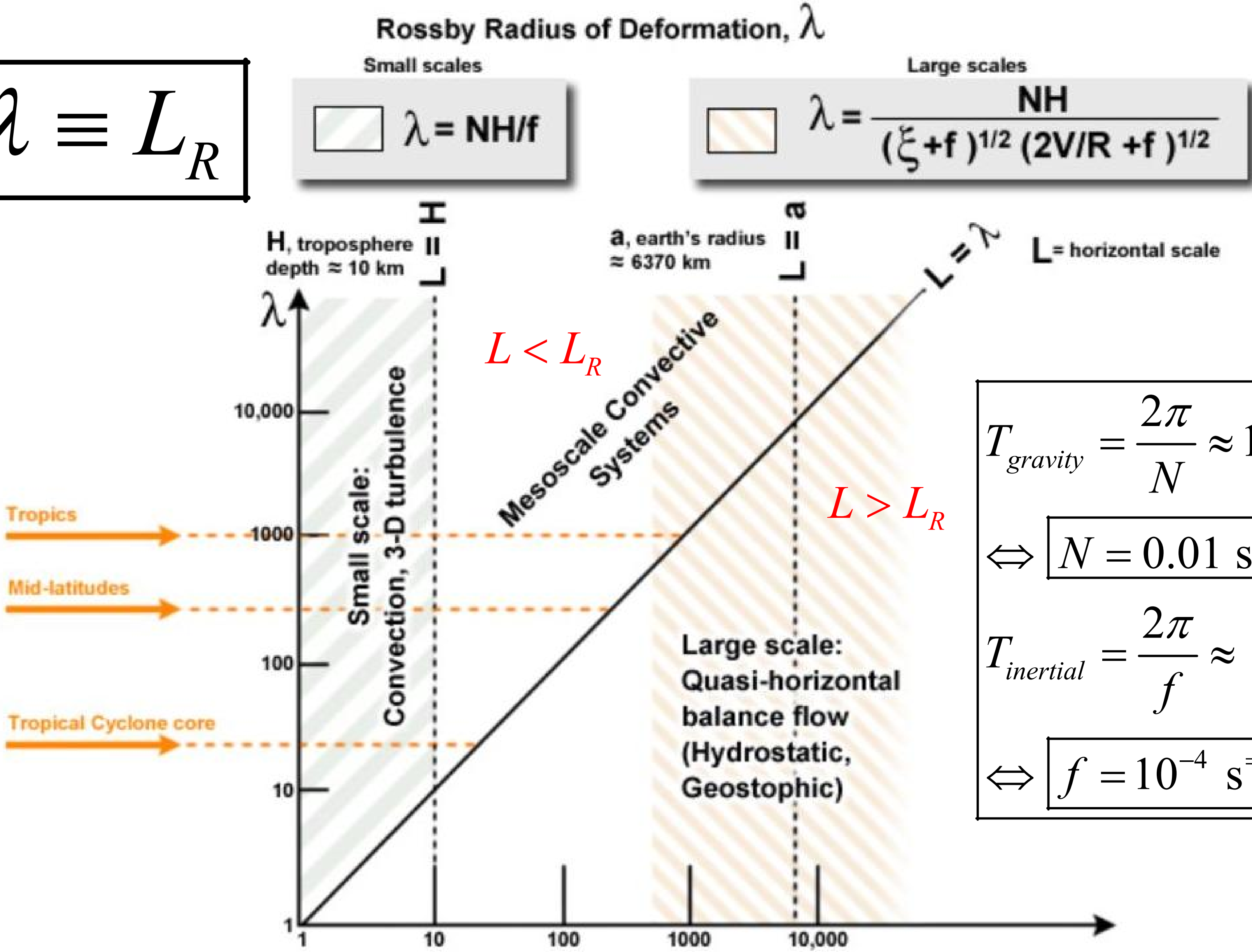
For a barotropic fluid, $L_R = \dfrac{\sqrt{gH}}{f}$

For a baroclinic fluid, $L_R = \dfrac{NH}{f}$

For a given fluid depth H, L_R increases with decreasing latitude for all H simply because a gravity wave can propagate further as the inertial period increases.
For a given f, L_R decreases with depth of the fluid, reflecting the decrease of gravity wave speed with the depth of the fluid. $\Leftrightarrow$ Clearly the choice of H is very important

$\Rightarrow$ L_R could be dominated by rotational processes at higher latitudes but by buoyancy effects in the tropics

$$\lambda \equiv L_R$$



$$T_{gravity} = \frac{2\pi}{N} \approx 10 \text{ min}$$

$$\Leftrightarrow \boxed{N = 0.01 \text{ s}^{-1}}$$

$$T_{inertial} = \frac{2\pi}{f} \approx 1 \text{ day}$$

$$\Leftrightarrow \boxed{f = 10^{-4} \text{ s}^{-1}}$$

$$\lambda = \frac{c_{gravity}}{f} = \frac{L_z N}{f} \approx \frac{NH}{f}$$

$$\lambda = c_{gravity} \Big/ \sqrt{(f+\zeta)\left(f + \frac{2v_{\tan}}{R}\right)}$$

⇒ L_R can be thought of as the length scale at which the velocity vector of a gravity wave is rotated such that it is perpendicular to the pressure gradient

⇒ Rossby Radius $L_R \approx \dfrac{NH}{f}$

⇒ In other words, L_R is the scale at which rotational effects become important.

⇒ For scales $L \approx L_R$, velocity and pressure fields both adjust to maintain balance between momentum and mass fields

⇒ $L >> L_R$, velocity field adjusts to the pressure field during geostrophic adjustment adjustment

⇒ $L << L_R$, pressure field adjusts to the velocity field during geostrophic adjustment

⇒ Synoptic scale $(\geq L_R)$ is characterized by near geostrophic balance for straight flow

→ Flow accelerations and ageostrophy are small.

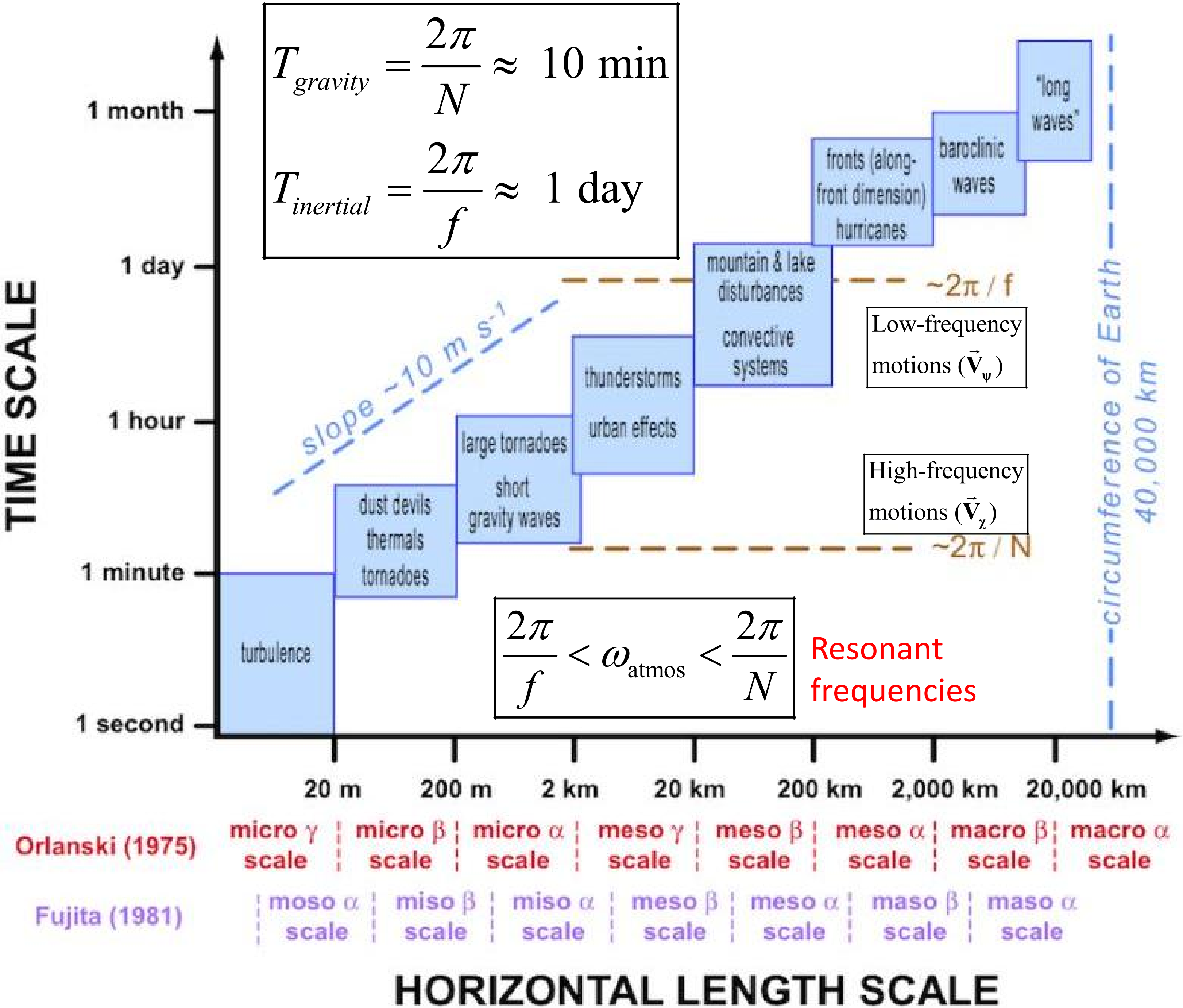
⇒ $L < L_R$ → pressure gradients can be much larger than on the synoptic scale while f remains of similar magnitude to that on the synoptic scale → large flow accelerations and ageostrophic motions → gradient wind balance doesn't hold.

Mid-latitude synoptic-scale motions are primarily driven by baroclinic instability.

⇒ Baroclinic instability most likely with disturbances $\approx 3 \times L_R$

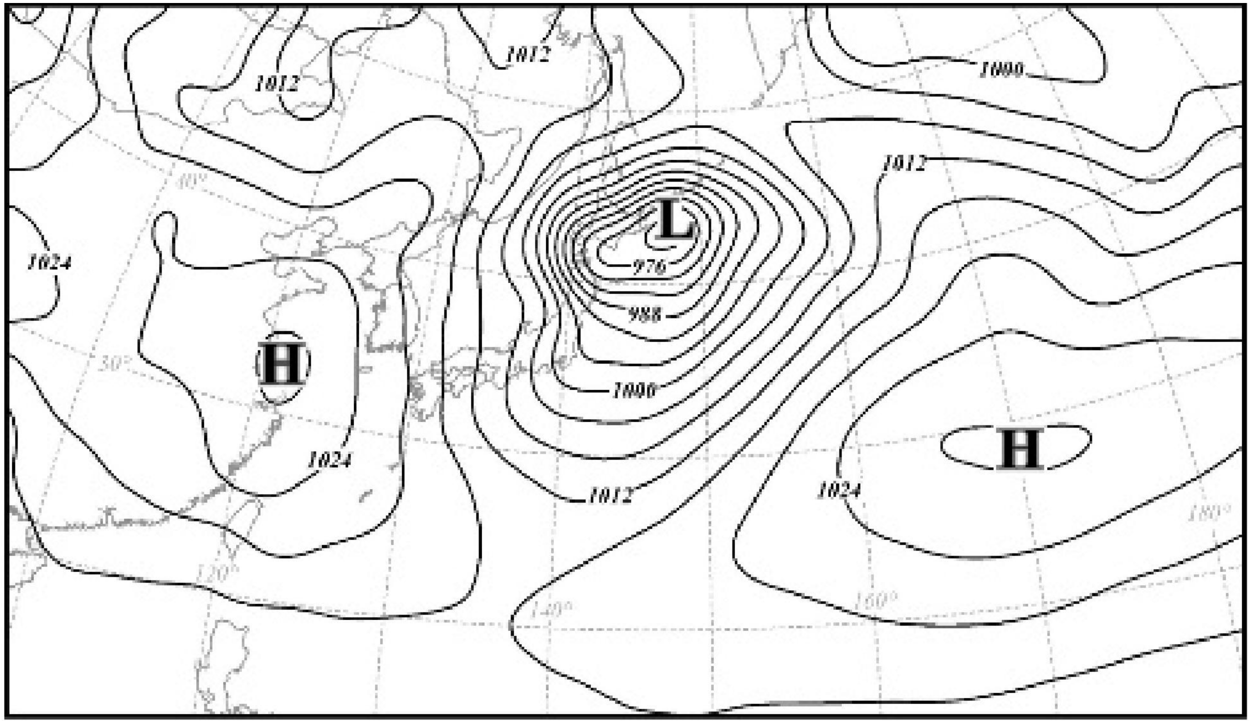
$$L_R = \frac{NH}{f} \approx \frac{0.01\ \text{s}^{-1} \times 10^4\ \text{m}}{10^{-4}\ \text{s}^{-1}} = 10^6\ \text{m} = 1000\ \text{km}$$

⇔ This is why $L = 1000$ km is used in the scale analysis



$$T_{gravity} = \frac{2\pi}{N} \approx 10 \text{ min}$$
$$T_{inertial} = \frac{2\pi}{f} \approx 1 \text{ day}$$
TIME SCALE
1 month
1 day
1 hour
1 minute
1 second
"long waves"
baroclinic waves
fronts (along-front dimension) hurricanes
mountain & lake disturbances
convective systems
~2π / f
Low-frequency motions ($\vec{\mathbf{V}}_\psi$)
thunderstorms
urban effects
slope ~10 m s-1
large tornadoes
short gravity waves
dust devils
thermals
tornadoes
High-frequency motions ($\vec{\mathbf{V}}_\chi$)
~2π / N
circumference of Earth
40,000 km
turbulence
$$\frac{2\pi}{f} < \omega_{\text{atmos}} < \frac{2\pi}{N}$$
Resonant frequencies
20 m
200 m
2 km
20 km
200 km
2,000 km
20,000 km
Orlanski (1975)
micro γ scale
micro β scale
micro α scale
meso γ scale
meso β scale
meso α scale
macro β scale
macro α scale
Fujita (1981)
moso α scale
miso β scale
miso α scale
meso β scale
meso α scale
maso β scale
maso α scale
HORIZONTAL LENGTH SCALE

Cyclones and anticyclones



Why anticyclones are larger in size than cyclones?

The Rossby radius can be scaled to $L_R = \dfrac{NH}{\zeta + f}$

L_R is larger (smaller) for an anticyclone (cyclone)

Geostrophic adjustment

- ***Suppose the winds and heights are in perfect geostrophic balance when a disturbance is suddenly imposed upon it.***
  - ***The atmosphere responds by sending out gravity wave pulses, spreading out like ripples on a pond when a rock is dropped. In the wake of these gravity waves is left behind a new geostrophically balanced state.***
  - ***The new state is different from the original geostrophic conditions in both the winds and heights. These changes remain for a long time because the new state is balanced. This process of the atmosphere evolving toward a balanced state is called "geostrophic adjustment."***

Geostrophic adjustment problem

To resolve this, consider the ways in which the mass and wind fields can adjust in such situations. The shallow-fluid equations represent a simple framework for addressing this, which nevertheless contains all the relevant dynamics. Two of the admissible wave solutions are gravity waves and inertia waves. Both mechanisms operate simultaneously to reconcile an imbalance.

The inertia waves modify the winds, and the gravity waves modify the mass field (the fluid depth, in the shallow-fluid system). As an indicator of how much of the adjustment results from changes in each of the mass field and momentum field, consider the periods of these waves.

For inertia waves , $T_{in} = \dfrac{2\pi}{f}$, and for gravity waves $T_{gr} = \dfrac{L}{\sqrt{gH}}$, where L is the length of the gravity wave (defined by the horizontal scale of the imbalance) and H is the depth of the imbalance (the vertical scale).

Geostrophic adjustment problem

Given that both types of waves simultaneously act to adjust the atmosphere toward the geostrophic state, the wave mode with the

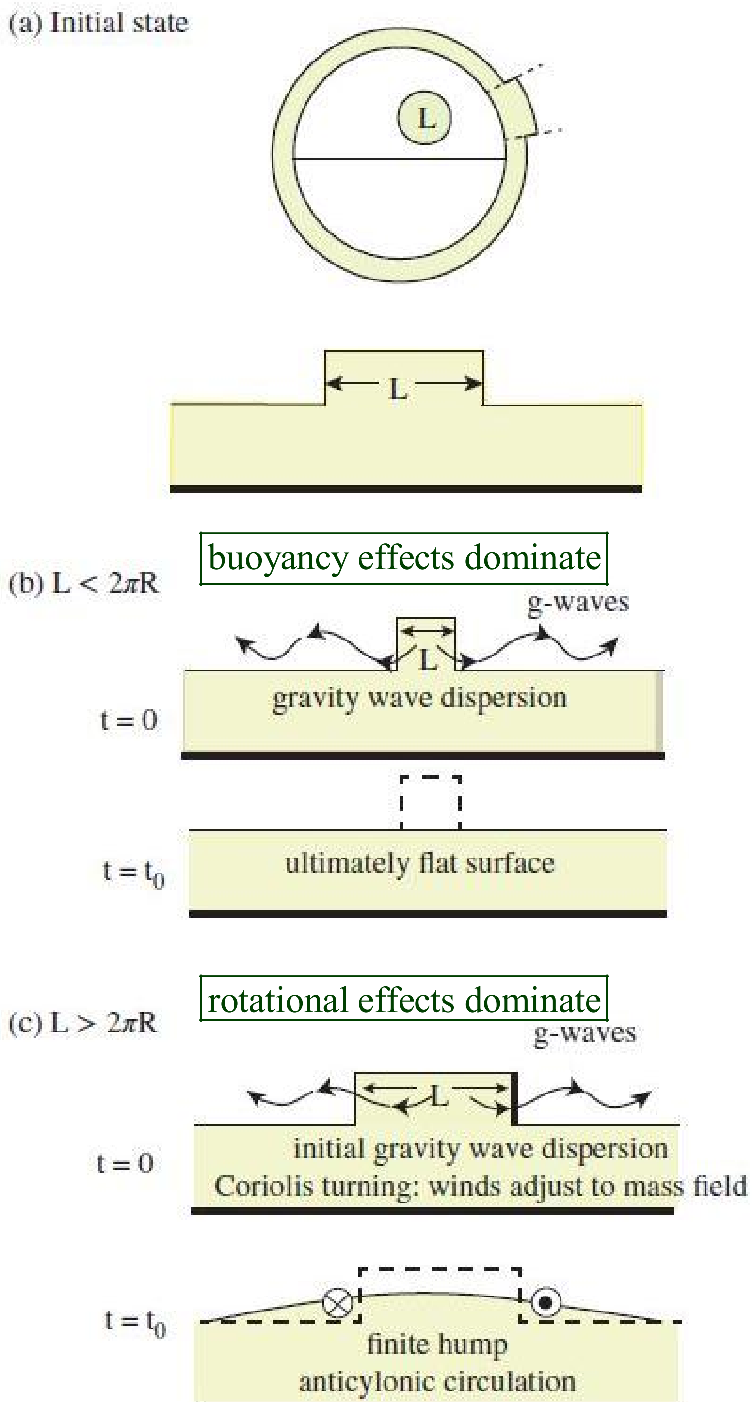
shortest period accomplishes most of the adjustment.

To define the condition where there is equal adjustment from both types of waves, the expressions for the two periods can be equated.

$$T_{in} = \frac{2\pi}{f}; \quad T_{gr} = \frac{L}{\sqrt{gH}}$$

Solving for the wavelength yields $\Leftrightarrow L_R = \dfrac{2\pi\sqrt{gH}}{f}$

L_R is the Rossby radius of deformation for the shallow-fluid system.
For wavelengths shorter than this value, redistribution of the mass field through gravity waves is responsible for most of the adjustment, whereas for longer wavelengths, modification of the windfield by the inertia waves accomplishes most of the adjustment.



Geostrophic adjustment

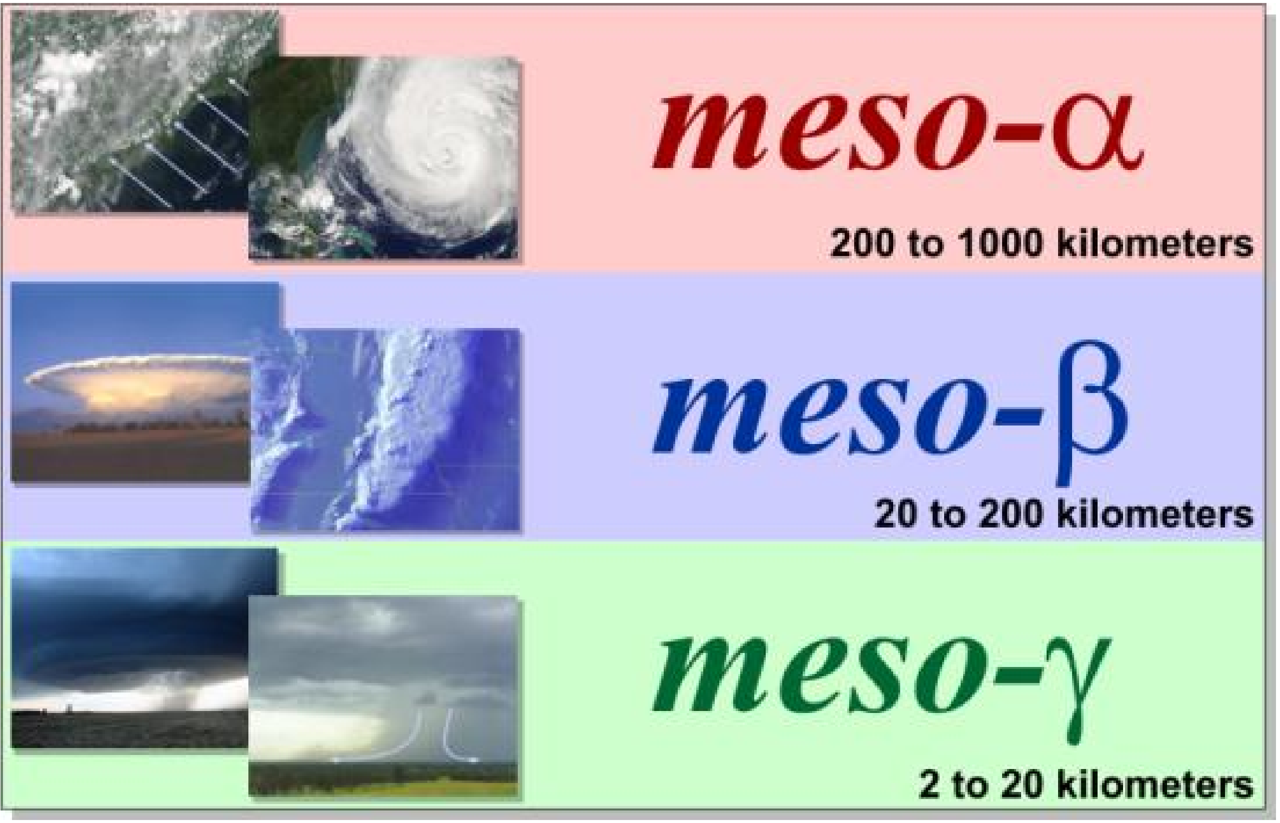
The Rossby Radius of Deformation, **R**, is the distance a gravity wave travels during one inertial period.

$L =$ Length scale of the disturbance

Case 1: If $\underline{L < 2.\pi R}$ then buoyancy effects dominate and gravity waves will disperse rapidly. The PE of the initial disturbance is quickly transformed into KE. Here the mass field adjusts to the wind field. For the smaller scale, there is no residual potential energy and the final state of the shallow fluid is flat.

Case 2: If $\underline{L > 2.\pi R}$ then rotational effects dominate and rotational waves are produced. For the larger scale case, there is sufficient time for the rotational factors to come into effect with the winds adjusting to the mass field. The final state is a finite hump with an anticyclonic circulation.

The limiting case is $\underline{L \approx 2\pi R}$ where mass and velocity fields mutually adjust with each other



Length scales $L \geq L_R$
Quasi-geostrophic
Wind adjustment

Length scales $L \leq L_R$
Inertia-gravity waves
Mass adjustment

Disturbances characterized by Gravity (Buoyancy) Waves (stable) or Deep Convection (unstable).

Scaling of horizontal motion in tropics

Equation for horizontal motion can be written as

$$\frac{\partial \vec{\mathbf{V}}}{\partial t} + \left(\vec{\mathbf{V}} \cdot \nabla\right)\vec{\mathbf{V}} + w\frac{\partial \vec{\mathbf{V}}}{\partial z} + f\hat{\mathbf{k}} \times \vec{\mathbf{V}} = -\frac{1}{\rho}\nabla p$$

$$\frac{U^2}{L} \qquad \frac{U^2}{L} \qquad \frac{WU}{H} \qquad fU \qquad \frac{\Delta p}{\rho L}$$

For midlatitudes ($\sim 45^0$N),

~~10^{-4}~~ ~~10^{-4}~~ ~~10^{-5}~~ [10^{-3} 10^{-3}]

For equatorial latitudes (5-10^0 latitude)

[10^{-4} 10^{-4}] ~~10^{-5}~~ [10^{-4}] [??]

For the balance to be maintained,

$$\text{For } L = 10^6 \text{ m}, \ \frac{\Delta p}{\rho L} \approx 10^{-4} \Rightarrow \Delta p \approx 10^2 = 1 \text{ hPa}$$

For equatorial zone (5^oS – 5^oN latitude)

~~10^{-5}~~ [10^{-4}] ~~10^{-5}~~ ~~10^{-5}~~ $\frac{\Delta p}{\rho L}$ $(\Delta p = 1 \text{ hPa})$

$\rightarrow$ PG force balanced by the advection term

Element	*Mid-latitudes*	*Tropical region*
U,V (horizontal velocity)	*10-20 m s^{-1}*	*10-20 m s^{-1}*
W (vertical velocity)	*1 cm s^{-1}*	*1 cm s^{-1}*
L (length, distance scale)	*1000 km*	*1000 km*
H (depth, height scale)	*10 km (depth of troposphere)*	*10 km*
Horizontal pressure change (Δp_H)	*10-20 hPa*	*1 hPa*
Vertical pressure change (Δp_V)	*1000 hPa*	*1000 hPa*
Time (L/U)	*27 hours*	*1 day*
ρ (density)	*1 kg m^{-3}*	*1 kg m^{-3}*
g (gravity)	*9.8 m s^{-2}*	*9.8 m s^{-2}*
Ω (angular velocity)	*7.292 × 10^{-5} s^{-1}*	*7.292 × 10^{-5} s^{-1}*

In tropical regions, the horizontal pressure gradients are <u>one order smaller</u> than in mid-latitudes

Why streamlines are good choice for tropics?

$$\frac{du}{dt}-\frac{uv\tan\varphi}{a}+\frac{uw}{a}=2\Omega\left(v\sin\varphi-w\cos\varphi\right)-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\nabla^2 u$$

Let $f = 2\Omega\sin\varphi$ and $f' = 2\Omega\cos\varphi$

$$\frac{du}{dt}-fv+f'w+\frac{uw}{a}-\frac{uv\tan\varphi}{a}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\nabla^2 u$$

$L=10^6$ m, $U,V = 10$ m s^{-1}, $H=10^4$ m, $\varphi=10^o$ (for simplicity), $f\approx 10^{-5}$ s^{-1}, W (small at large scales) = 10^{-2} m s^{-1}, $a =$ Radius of the earth $\approx 10^7$ m, $\rho =$ Air density = 1 kg m^{-3}. ν = dynamic viscosity of air $\approx 10^{-5}$ m^2 s^{-1} $\rightarrow$ $\nu\nabla^2 u\approx 10^{-12}$ m s^{-2} (tiny)

$$\frac{U^2}{L}-fU+f'W+\frac{UW}{a}-\frac{UV\tan\varphi}{a}=-\frac{1}{\rho}\frac{\delta p}{L}+F \quad\rightarrow\quad \frac{V^2}{R}+fV=-\frac{1}{\rho}\frac{\partial p}{\partial R}$$

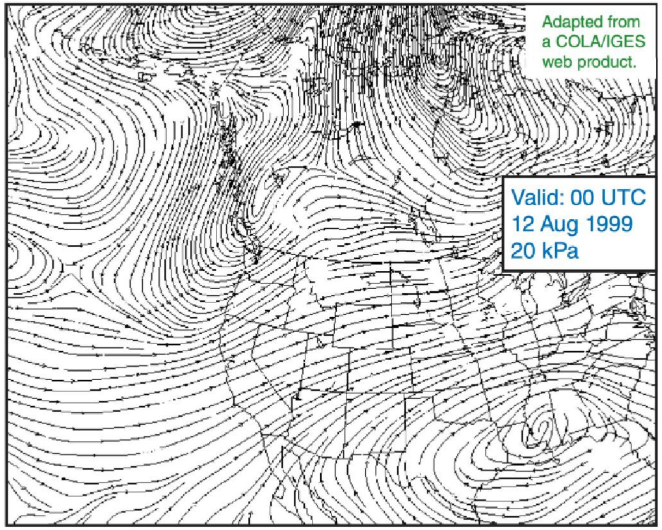
$$10^{-4}\quad 10^{-4}\qquad 10^{-6}\qquad 10^{-8}\qquad 10^{-6}\qquad 10^{-4}\qquad 10^{-12}$$

$\Rightarrow$ This leaves us with gradient wind balance on synoptic length scales in tropics, since f is small.

$\Rightarrow$ Pressure gradient is about 100 Pa per 1000 km (i.e., 1 hPa for a length scale of L = 1000 km).

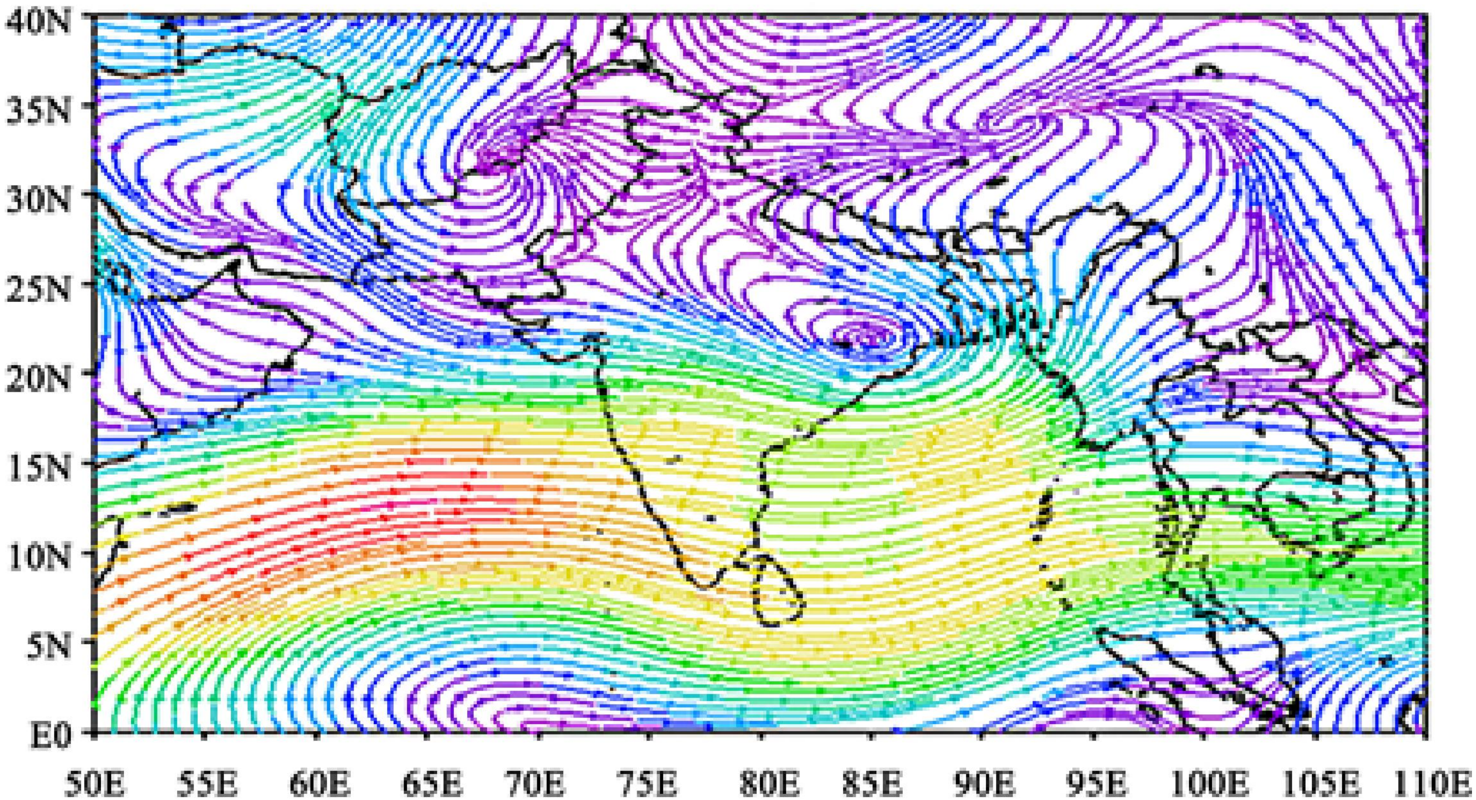
$\Rightarrow$ Since Coriolis acceleration is weak, synoptic scale pressure gradients are small, plotting surface pressure gradients in the tropics is not a particularly useful analysis in general.

$\Rightarrow$ Convergence and rotation of winds are important to tropical circulations. Therefore, streamline analysis is of greater use than plotting pressure on a constant height surface especially at low levels, i.e., for better description of confluence (diffluence) is when streamlines come together (move apart)



⇒ Convergence and rotation of winds are important to tropical circulations. Therefore, streamline analysis is of greater use than plotting pressure on a constant height surface especially at low levels, i.e., for better description of confluence (diffluence) is when streamlines come together (move apart)

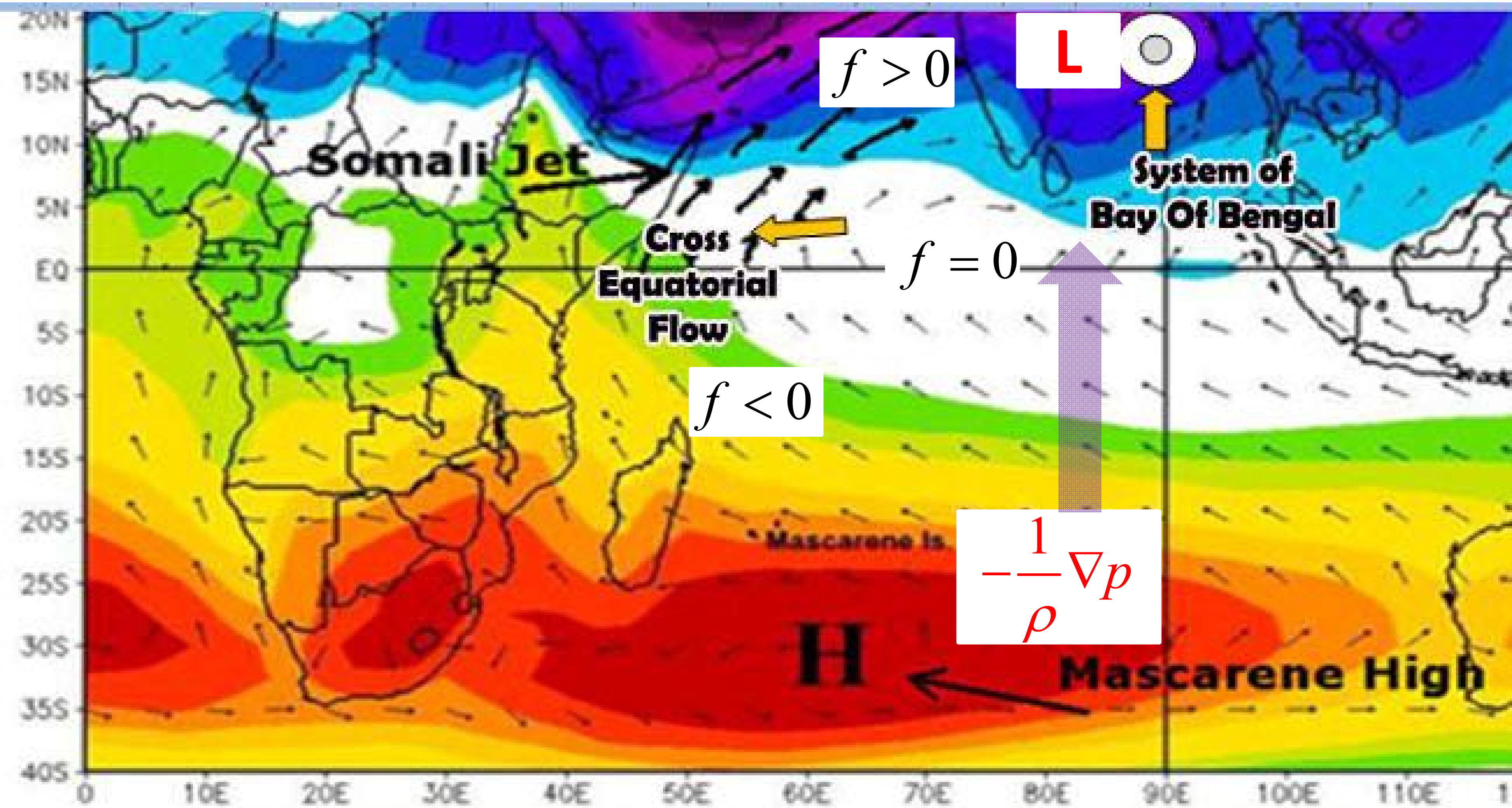
Synoptic motion scales in tropics: $L \approx 1000$ km, $\Delta p \approx 1$ hPa, $\Delta T \approx 0.3^o$C



Indian monsoon context

⇒ quasi-geostrophic theory applicability

⇒ small Ro flows for planetary scale



⇒ monsoon circulation to be viewed on planetary scale ($L \approx a$ = radius of the earth = 6378 km)

$$Ro = \frac{U}{fL} = \frac{10 \text{ m s}^{-1}}{10^{-5} \text{ s}^{-1} \times 10^{7} \text{ m}} = 0.01$$

Temperature and pressure gradients in the tropics

Synoptic scale temperature gradients (like pressure gradients) in the tropics are small

The hydrostatic equation can be written as

$$\frac{\partial \Phi}{\partial p} = -\frac{RT}{p} = -\frac{1}{\rho} \approx -1 \Rightarrow \text{Since } \rho \approx 1 \text{ kg m}^{-3}$$

Following ideal gas equation, $\Delta p = \rho R \Delta T$

$$\Delta T\big|_{\text{tropics}} \approx \frac{\Delta p}{R} \approx \frac{1 \text{ hPa}}{R} \approx 0.35 \text{ K}$$

$$\Delta T\big|_{\text{midlatitudes}} = \frac{10 \text{ hPa}}{R} \approx 3.5 \text{ K}$$

$$\frac{\delta(p, \rho, \theta)}{(p, \rho, \theta)} \equiv \frac{Fr^2}{Ro}, \text{ where } Fr = \frac{U}{\sqrt{gH}}; \text{ Ro} = \frac{U}{fL}$$

$$Ro << 1 \Rightarrow \frac{\delta p}{p} \approx \frac{\delta \rho}{\rho} \approx \frac{\delta \theta}{\theta} \approx 10^{-2} \Leftrightarrow \text{mid-latitudes } (Fr \approx 10^{-3}; Ro \approx 0.1)$$

$$Ro \approx 1 \Rightarrow \frac{\delta p}{p} \approx \frac{\delta \rho}{\rho} \approx \frac{\delta \theta}{\theta} \approx 10^{-3} \Leftrightarrow \text{tropics } (Fr \approx 10^{-3}; Ro \approx 1)$$

For scales of $L = 10^6$ m, the fluctuations in p, ρ and θ are an order of magnitude smaller in the tropics than in middle latitudes.

Is the Tropical Atmosphere Hydrostatic?

$$\frac{dw}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g + 2\Omega u\cos\varphi + \frac{u^2+v^2}{r_e}$$

$$|Wf_r| \qquad \left|\frac{P}{\rho D}\right| \qquad |g| \qquad |\Omega U| \qquad \left|\frac{U^2}{a}\right|$$

$U = 10\ \text{m s}^{-1}$; $\Omega \approx 10^{-5}\ \text{s}^{-1}$; $a \approx 6400$ km

$W \equiv 1\ \text{cm s}^{-1}$; $L = 10^6$ m ; $\tau = \frac{L}{U}$; $\frac{1}{\tau} \equiv f_r \equiv \frac{U}{L}$

For high-frequency limit: $f_r >> f \Rightarrow \rho LUf_r >> \rho LUf \Rightarrow$ $P = P_1 = \rho LUf_r$ $\Leftrightarrow$ $f\tau << 1$

For low-frequency limit: $f_r << f \Rightarrow \rho LUf_r << \rho LUf \Rightarrow$ $P = P_2 = \rho LUf$ $\Leftrightarrow$ $f\tau >> 1$

For hydrostatic balance, $\frac{dw}{dt}\Big/\frac{1}{\rho}\frac{\partial p}{\partial z} = S << 1$

For high-frequency limit: $\Rightarrow \left(\frac{WU}{L}\right)\Big/\left(\frac{P_1}{\rho D}\right) = \frac{W}{U}\left(\frac{D}{L}\right) \Leftrightarrow$

Inertial motions are hydrostatic
if $W << U,\ \ D << L \ \Leftrightarrow S << 1$
$\rightarrow$ gravity waves treated hydrostatically

For low-frequency limit: $\Rightarrow \left(\frac{WU}{L}\right)\Big/\left(\frac{P_2}{\rho D}\right) = \frac{W}{U}\left(\frac{D}{L}\right)\left(\frac{f_r}{f}\right) \Leftrightarrow$

Even if $W \approx U,\ \ D \approx L$
$\rightarrow$ hydrostatic approximation is
justified, as long as $f_r << f \ \Leftrightarrow S << 1$

For synoptic ($L = 10^6$ m) or planetary scales ($L \approx a$), both of which have scales $L >> D$, the hydrostatic approximation remains valid even if $f_r \approx f$, as f decreases toward the equator.
$\rightarrow$ Thus, the application of the hydrostatic approximation for all motions of scales greater than cloud clusters appears justified.

Synoptic scale vorticity dynamics (tropics)– Scale analysis

$$\frac{\partial \zeta}{\partial t} + \vec{\mathbf{V}}_{\mathbf{H}} \cdot \nabla(\zeta + f) + w\frac{\partial \zeta}{\partial z} = -(\varsigma + f)\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) - \left(\frac{\partial w}{\partial x}\frac{\partial v}{\partial z} - \frac{\partial w}{\partial y}\frac{\partial u}{\partial z}\right) + \frac{1}{\rho^2}\left(\frac{\partial \rho}{\partial x}\frac{\partial p}{\partial y} - \frac{\partial \rho}{\partial y}\frac{\partial p}{\partial x}\right)$$

$$\frac{U^2}{L^2} \qquad \frac{WU}{HL} \qquad \frac{f_0 U}{L} \qquad \frac{WU}{HL} \qquad \frac{\delta p \delta \rho}{\rho^2 L^1}$$

$$\boxed{10^{-10}} \qquad 10^{-11} \qquad 10^{-10} - 10^{-11} \qquad 10^{-11} \qquad 10^{-11}$$

On the synoptic scale tropical motions, the vorticity equation can be

can be approximated as $\boxed{\frac{\partial \zeta}{\partial t} + \vec{\mathbf{V}}_{\mathbf{H}} \cdot \nabla(\zeta + f) = 0} \Leftrightarrow$ Non-divergent

It tells us that outside regions where condensation are important, not only the vertical motion is exceedingly small, and the flow is almost non-divergent (barotropic/no divergence in the large-scale tropical flow)

such motions cannot generate KE from PE, they must obtain their energy either from barotropic processes (such as barotropic instability mechanisms)

How can the kinetic energy be generated in tropics?

$\Leftrightarrow$ Answer lies in convective processes

$U = 10 \text{ m s}^{-1}$

$W = 1 \text{ cm s}^{-1}$

$L = 1000 \text{ km} = 10^6 \text{ m}$

$H_{\text{Troposphere}} = 10 - 20 \text{ km}$

$\boxed{\Delta p = 1 \text{ hPa}}_{\text{Tropics}} \Downarrow$

$\rho = 1 \text{ kg m}^{-3}$

$f_{5-30^0 N} \approx 10^{-5} \text{ s}^{-1} \Downarrow$

$\beta = \frac{2\Omega}{a} \approx 2 \times 10^{-11} \text{ m}^{-1}\text{s}^{-1}$

In tropics, the flow is almost non-divergent on synoptic scale

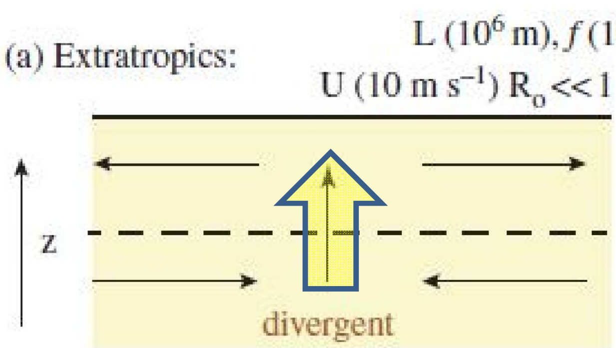
Remember: Vorticity equation for large-scale motions

$$\frac{d}{dt}(\varsigma + f) = -(\varsigma + f)\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) \Leftrightarrow \text{For extratropics, } L = 1000 \text{ km, } Ro << 1$$

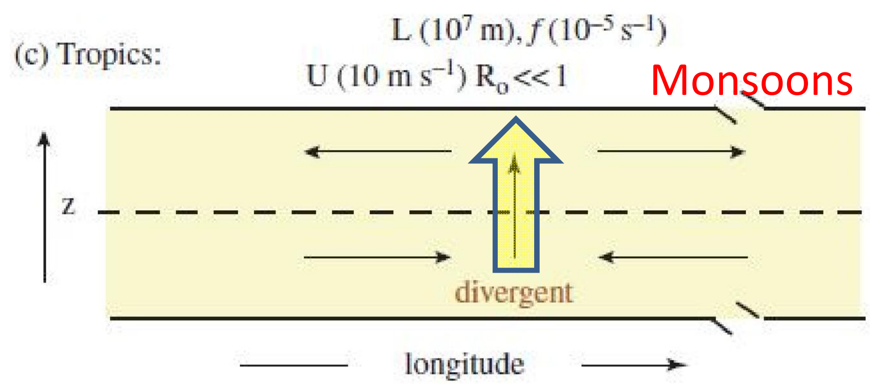
$\Rightarrow$ Here the flow is baroclinic, right-hand term suggests large-scale vertical communication

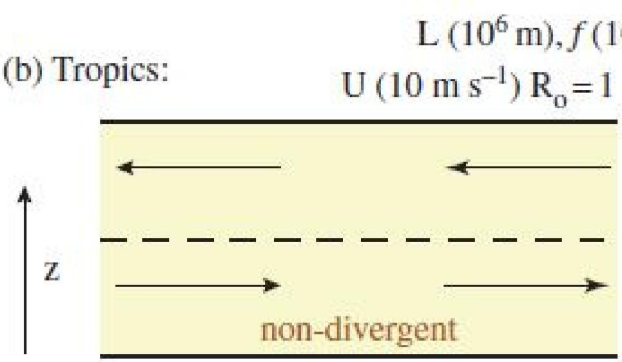
$$\frac{d}{dt}(\varsigma + f) = 0 \quad \Leftrightarrow \text{For tropics, } L = 1000 \text{ km, } Ro \approx 1$$

$\Rightarrow$ vorticity equation for 10^6 m scales of motion in the tropics would appear to be predominantly barotropic. However, note that for larger scales of motion in the tropics ($L > 10^7$ m), the form adopts a divergent form $\rightarrow$ For tropics, $L = 10000$ km, $Ro << 1$



communication between the lower and upper troposphere in terms of divergence.





Non-divergent nature of tropical motions of the same spatial scale and <u>a lack of communication between the lower and upper troposphere</u>.

($L \sim 10^7$ km) : Walker and Hadley circulations and the major monsoon systems, tropical motions are <u>divergent</u>

On scales of $L \sim 10^6$ m, tropical motions appeared to be non-divergent
On scales of $L \sim 10^7$ m, tropical motions are moderately divergent

Extratropics vs Tropics

	$\frac{d\vec{\mathbf{V}}}{\partial t}$	$f\hat{\mathbf{k}} \times \vec{\mathbf{V}}$	$-\frac{1}{\rho}\nabla p$	Ro	
Extratropics	~1	~10	~10	~0.1	Geostrophic
Tropics	~1	~1	~1	~1	Gradient

- In the extratropics, the weather is controlled by migratory systems in the form of extratropical waves and quasi-stationary systems
- On the other hand, generally the migratory and quasi-stationary systems in the tropics are weak.

Distinguishing between large-scale and mesoscale

- *Large-scale (synoptic and sub-synoptic) processes can be restricted to:*
  - *Adiabatic*
  - *Hydrostatic*
  - *Mass continuity must be satisfied*
  - *Advection is dominated by the geostrophic wind*
- *Mesoscale (thermodynamic environment) stands in between large and small scales*
  - *Defined as processes which cannot be understood without considering the large scale and microscale processes*

Rossby radius

- ***RROD is defined as the radius at which rotation becomes as important for maintenance of a circulation as buoyancy.***
- ***The basic premise is that small circulations are typically dominated by buoyancy forcing, which results in gravity waves quickly dispersing energy in a stable environment. Thus, most of the energy is released as kinetic energy.***
- ***Larger circulations are more rotational in character, and are dominated by rossby wave dynamics, allowing for additional persistence. In this scenario, potential energy is stored within the circulation.***
- ***Simply put, if a disturbance is larger (smaller) than the environmentally derived RROD, it will persist (dissipate)***

$$L_R = \frac{NH}{\zeta + f}$$